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Service design of consumer data intermediary for competitive individual targeting

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A B S T R A C T

Individual targeting, a marketing strategy that firms target individual consumers with tailored offers, is currently a widespread practice. Customer data intermediaries (CDIs) have emerged recently to help firms learn their prospective customers and launch their target marketing campaigns. This paper uses a common-value auction framework to study how a CDI designs and differentiates its information services to help two competing firms identify and target valuable customers. We characterize the firms' equilibrium target marketing strategies. The results show that the CDI serves one firm exclusively in unpromising markets where the proportion of valuable customers is relatively low, and provides both firms with differentiated services in promising markets where the proportion of valuable customers is relatively high. In addition, the CDI differentiates its services less when the proportion of valuable customers is higher.

1. Introduction

The past decade has witnessed the boom of target marketing based on consumer data [10,11]. For example, retailers provide checkout coupons to consumers based on their past purchases. Phone carriers (e.g., AT&T) often lure customers by offering them personalized checks depending on consumers' calling history. Credit card companies (e.g., American Express) usually issue preapproved credit cards to customers with good credit scores. Banks (e.g., Bank of American) often promote diverse financial products with free trials based on their clients' account information.

Customer data intermediaries (CDIs) have emerged to help firms implement target marketing. CDIs collect individual-level customer data through various online and offline channels and facilitate firms to target individual customers with tailored advertisements or promotional incentives. For example, Catalina Marketing discovers consumers' shopping habits using data collected through its network of 24,000 U.S. and 8000 international merchandiser stores, and provides its clients various target marketing services, such as checkout coupons, quick cash and shopping lists.[1] DoubleClick tracks Web users by name and address as they move from one website to the next, and helps firms deliver targeted banner advertisements.[2] Pancras and Sudhir [15] summarize major CDIs and their services. The CDI services significantly improve the effectiveness of target marketing campaigns. For example, the average redemption rate of Catalina Marketing incentives (coupons) is about 6.3%, more than eight times greater than other non shopper-driven traditional promotional methods.[3] As firms compete aggressively using target marketing, CDIs play an important role in shaping the firms' competition.

Considering the growing importance of CDIs, there is a need to study CDIs' businesses from strategic perspectives. In different markets, CDIs employ significantly diverse business strategies [15]. Some CDIs offer their services on an exclusive basis and others offer on a nonexclusive basis. For example, Catalina only sells its services exclusively, but Abacus and I-Behavior sell on a nonexclusive basis to any catalog marketer or specialty retailer. It is important to understand what drives such diversity in CDIs' businesses. In addition, when the CDIs offer nonexclusive services, the CDIs often offer services with differential levels of service differentiation. Therefore, it is worthwhile to examine how the CDI's service differentiation influences market competition and how the CDI makes strategic decisions on service differentiation.

The purpose of this paper is to examine the impact of CDI services on firm competition and explain the diversity of CDIs' business strategies. We develop a model in which a CDI may help two competing firms identify and target valuable prospective customers. Compared to the firms, the CDI can better segment prospective customers based on the data it collects. With the CDI's services of customer segmentation, firms can better tailor promotional incentives to different customer segments. Using this model, we examine the CDI's selling strategy of its services. In particular, when should the CDI offer the service to only one firm exclusively and when should the CDI serve both firms? If the CDI serves both firms, how should it differentiate its services between firms? Because the CDI's service strategies influence the firms' competition, our model also characterizes how firms compete using target marketing.

The existing research on firm competition with target marketing largely focuses on horizontally differentiated firms (e.g., [3,4,8,9,12,13,16,17]). The individual-level customer information is used to

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[1] http://www.catalinamarketing.com/company/catalina_quick_stats.html.

[2] DoubleClick was acquired by Google in 2007. Since 2008 Google controls estimated 69% of the online advertising market.

[3] http://www.catalinamarketing.com/company/FAQs.html

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discover the customers' brand preferences (i.e., horizontal preferences). This study, in contrast, considers the individual-level information about customers' heterogeneous preferences to the product itself (i.e., vertical preferences). Our model captures the competition between two firms selling identical products. Therefore, the valuable customers are equally valuable to both firms, and the firms' competition exhibits the features of common-value auction (e.g., [6,14]). The focus on the customer's product preference, instead of the brand preference, enables us to gain useful insights that are not captured in the existing literature. For example, the existing literature generally suggests that the decrease in information asymmetry among firms softens their competition for each other's loyal customers (who are valuable to only one firm), but intensifies their competition for comparison shoppers (common-value customers) when firms can better distinguish comparison shoppers from loyal customers (e.g., [3]). However, our study reveals that the decrease in information asymmetry among firms does not always intensify their competition for common-value customers.

This study also relates to the research on quality differentiation of information product/service. The existing literature has well explored this issue from the standpoints of monopolist sellers (e.g., [1,19]) or competing sellers (e.g., [5,21]). However, quality differentiation by the information intermediary has received relatively less attention. Bhargava and Choudhary [2] show that an information intermediary can provide quality-differentiated services and benefit from positive cross-network effects. Weber and Zhang [22] consider how a search intermediary provides differentiated paid referral services through the design of search ranking. These models, in line with most of traditional quality differentiation models, assume that the users have heterogeneous preferences to product quality. In contrast, our model considers the case that firms (i.e., users) are *ex ante* homogeneous and the firm heterogeneity is completely endogenized by the intermediary (i.e., the CDI) through service differentiation. Our model illustrates a different strategic effect of quality differentiation, i.e., the CDI's service design influences the competition between homogeneous firms in target marketing. A similar issue has been explored in the studies on referral infomediaries (e.g., [4,7]). These models, however, only consider the use of service exclusivity as a differentiation approach. Our model also considers the use of non-exclusive services with differentiated information quality as a differentiation approach. In addition, this study also explores how market conditions influence CDIs' business strategies. Our model considers both the promising markets where the majority of prospective customers are valuable customers and the unpromising markets where the majority of prospective customers are non-valuable customers. The analysis characterizes how the CDI's service strategies are dependent on the market conditions. The results help explain the diversity of CDIs' strategies across different markets, and also provide managerial implications on the CDI businesses in different markets.

The findings of this study are as follows. The study finds that when the CDI provides firms more similar information about the common-value customer (i.e., less differentiated services), the firms' competition may be softened. Although some prior studies (e.g., [3]) also suggest that information similarity may soften firm competition, their analyses are based on the models with *ex ante* differentiated firms. Our model generates this finding in the setting where firms are *ex ante* homogeneous and the firm differentiation is completely endogenized by the CDI. Our model also characterizes when the decrease in information asymmetry between firms softens the competition and when it intensifies the competition. This insight generates important implications for the CDI's service strategies. When the CDI serves both firms, adjusting the service differentiation between firms allows the CDI to maintain a desirable level of firm competition and maximize its profit from selling information services to firms. Based on the results of firm competition, the study characterizes the optimal service strategies for the CDI. The analysis suggests that in unpromising markets where the proportion of valuable customers is relatively small (i.e., the proportion of valuable customers is lower than $\sqrt{3}/3$), the CDI serves only one firm exclusively. However, in promising markets where the proportion of valuable customers is relatively large (i.e., the proportion of valuable customers is higher than $\sqrt{3}/3$), the CDI serves both firms but provides differentiated services. The study also characterizes the optimal degree of service differentiation.

The rest of the paper is organized as follows. Section 2 explains the model setup. Section 3 examines the firms' competition and Section 4 characterizes the service strategies of the CDI. Section 5 concludes the paper.

2. Model setup

We model a case where two firms, firm 1 and 2, compete in acquiring a group of prospective customers. A prospective customer, once acquired by a firm, can bring the firm sales revenue over her lifetime business relationship with the firm. We assume that there are two types of customers: valuable customers and non-valuable customers. We use V to denote the customer value. A valuable customer, once acquired by a firm, can generate a positive amount of revenue $R>0$ for the firm. Therefore, the value of a valuable customer is $V=R$ for the firm. A non-valuable customer, in contrast, does not generate any revenue for the firm, i.e., $V=0$. We let $T\in\{H,L\}$ denote the type of a customer with $T=H$ representing a valuable customer and $T=L$ representing a non-valuable customer. λ is used to denote the prior probability that a prospective customer is valuable, and $0<\lambda\leq 1$.

Firms compete for prospective customers by offering them promotional incentives. Examples of promotional incentives include target coupons, price discounts or other nonmonetary benefits. We use m_i ($i\in\{1,2\}$) to represent the monetary value of firm i's targeted promotional incentives. In this model, it is assumed that the two firms sell identical products/services and the prospective customers have no brand preference. The customers are acquired by the firm which offers them the higher promotional incentives. This also implies that the value of customers is the same to both firms. Therefore, our model exhibits the features of common-value auctions (e.g., [6,14]) and distinguishes from the traditional models of target marketing competition (e.g., [3,4]). Without loss of generality, the model can be simplified as that the two firms compete for a representative prospective customer who is valuable with a prior probability of λ.

We assume that firms do not directly observe whether or not the prospective customer is valuable. A CDI can help firms identify the customer value and improve their targetability. The CDI, armed with vast individual-level customer data collected from multiple sources, is more capable to estimate the value of the prospective customer. For example, if the data shows that the customer has been active in purchasing over the past few months, this customer is more likely to be a valuable customer. Otherwise, the customer is more likely to be a non-valuable customer. We model the case where the CDI provides customer segmentation services to firms. Specifically, the CDI classifies the customer into one of two segments, a *high segment* for valuable customer or a *low segment* for non-valuable customers. The CDI's customer segmentation service generates value by allowing the firm to be more informative about the customer value. That is, if the firm observes that the customer is classified into the high (low) segment, it should become more confident that this customer is valuable (non-valuable). With the CDI's service of customer segmentation, the firm can customize its promotional incentives sent to each segment.

We use S to denote the customer segmentation service that the CDI provides to firm 1. S has two potential values, h and l. $S=h$ means that the CDI classifies the customer into the high segment for firm 1. $S=l$ means that the CDI classifies the customer into the low segment for firm 1. We use $Pr(h)$ and $Pr(l)$ to denote the probability that the customer is classified into the high segment (i.e., $S=h$) for firm 1 and the probability that the customer is classified into the low segment (i.e., $S=l$) for firm 1, respectively.

Note that the customer segmentation S may not be completely accurate. That is, it is possible that the valuable (non-valuable) customer is incorrectly classified into the low (high) segment. We use ψ to denote the probability that the valuable customer is correctly classified into the high segment for firm 1, and use ϕ to denote the probability that the non-valuable customer is incorrectly classified into the high segment for firm 1. Therefore, the CDI can increase the information accuracy of the customer segmentation S by increasing ψ and decreasing ϕ, e.g. the CDI can analyze more historical data and/or apply more strict criteria to select valuable customers. We assume that

$$\psi > \phi. \tag{1}$$

The assumption (1) ensures that the segmentation S provides useful information for firm 1. To see that, note that we have

$$Pr(h) = \lambda\psi + (1-\lambda)\phi \text{ and } Pr(l) = \lambda(1-\psi) + (1-\lambda)(1-\phi). \tag{2}$$

Using the Bayes rule, we can derive the following posterior beliefs.

$$P(H|h) = \frac{\lambda\psi}{Pr(h)}, P(L|h) = 1-P(H|h),$$
$$P(L|l) = \frac{(1-\lambda)(1-\phi)}{Pr(l)}, P(H|l) = 1-P(L|l).$$

where $P(H|h)$ ($P(L|h)$) is the posterior probability that the prospective customer is a valuable (non-valuable) customer given that the customer is classified into the high segment (i.e., $S=h$) for firm 1, and $P(L|l)$ ($P(H|l)$) is the posterior probability that the prospective customer is a non-valuable (valuable) customer given that the customer is classified into the low segment (i.e., $S=l$) for firm 1.[4] The assumption (1) ensures the following inequalities hold.

$$P(H|h) > \lambda \text{ and } P(L|l) > 1-\lambda. \tag{3}$$

Inequalities (3) indicate that the posterior probability of a high-segment (low-segment) customer being valuable (non-valuable) is higher than the prior probability, λ $(1-\lambda)$. Therefore, the CDI's segmentation provides firm 1 with useful information about the customer's value. We use $E[V|S]$ ($S\in\{h,l\}$) to denote the expected customer value for firm 1 given S.

$$E[V|h] = RP(H|h) \text{ and } E[V|l] = RP(H|l).$$

In addition to firm 1, the CDI decides whether or not to serve firm 2. If the CDI serves both firms, it may differentiate its services. For example, if the CDI uses less data and less sophisticated business intelligence techniques in serving firm 2, the customer segmentation for firm 2 can be less accurate than that for firm 1. To capture the potential service differentiation, we use $\tilde{S}$ to denote the customer segmentation for firm 2. $\tilde{S}$ has two potential values, $\tilde{h}$ and $\tilde{l}$. $\tilde{S} = \tilde{h}$ ($\tilde{S} = \tilde{l}$) means that the CDI classifies the prospective customer into the high (low) segment for firm 2. Without loss of generality, we assume that when the CDI differentiates services, the customer segmentation S for firm 1 is always more accurate than the customer segmentation $\tilde{S}$ for firm 2. Specifically, we use $Pr\left(\tilde{h}|S\right)$ and $Pr\left(\tilde{l}|S\right)$ to denote the probabilities of $\tilde{S} = \tilde{h}$ and $\tilde{S} = \tilde{l}$ respectively, conditional on the value of S. $Pr\left(\tilde{h}|S\right)$ and $Pr\left(\tilde{l}|S\right)$ are defined as follows.

$$Pr\left(\tilde{h}|h\right) = Pr\left(\tilde{l}|l\right) = \frac{1+\zeta}{2}, Pr\left(\tilde{l}|h\right) = Pr\left(\tilde{h}|l\right) = \frac{1-\zeta}{2}. \tag{4}$$

where $\zeta\in[0,1]$ is the *similarity factor*. It captures to what extent the segmentation $\tilde{S}$ is similar to the segmentation S.[5]

If $\zeta=1$, the segmentation $\tilde{S}$ is the same as S. In this case, the CDI's services to both firms are not differentiated. If $0<\zeta<1$, when $S=h$, it is more likely that $\tilde{S} = \tilde{h}$. In other words, when a customer is classified into the high segment for firm 1, she is more likely to also be classified into the high segment for firm 2. Similarly, when $S=l$, it is more likely that $\tilde{S} = \tilde{l}$. However, as long as $\zeta<1$, the segmentation $\tilde{S}$ is less accurate than the segmentation S. When $\zeta=0$, given any value of S, we have $\tilde{S} = \tilde{h}$ or $\tilde{l}$ with 1/2 probability. This essentially means that the segmentation $\tilde{S}$ provides no useful customer information to firm 2. Therefore, we can consider $\zeta=0$ as the case that the CDI does not serve firm 2 (i.e., the CDI serves firm 1 exclusively), and firm 2 just uses the prior probability to estimate the customer value.

Given a $\tilde{S}\in\left\{\tilde{h},\tilde{l}\right\}$, the expected value of the customer to firm 2 can be represented as

$$E\left[V|\tilde{S}\right] = P\left(h|\tilde{S}\right)E\left[V|h\right] + P\left(l|\tilde{S}\right)E\left[V|l\right], \tag{5}$$

where $P\left(h|\tilde{S}\right)$ and $P\left(l|\tilde{S}\right)$ are posterior probabilities of $S=h$ and $S=l$ respectively, given a $\tilde{S}\in\left\{\tilde{h},\tilde{l}\right\}$. Using the Bayes' rule, we have

$$P\left(h|\tilde{S}\right) = \frac{Pr\left(\tilde{S}|h\right)Pr(h)}{Pr\left(\tilde{S}|h\right)Pr(h) + Pr\left(\tilde{S}|l\right)Pr(l)} \text{ and} \tag{6}$$
$$P\left(l|\tilde{S}\right) = \frac{Pr\left(\tilde{S}|l\right)Pr(l)}{Pr\left(\tilde{S}|l\right)Pr(l) + Pr\left(\tilde{S}|h\right)Pr(h)}.$$

It is worth noting that we have $P\left(T|\tilde{S}\right) = Pr(T|h)P\left(h|\tilde{S}\right) + Pr(T|l)P\left(l|\tilde{S}\right)$, where $T=\{H,L\}$. This suggests that $P\left(H|\tilde{h}\right) \le P(H|h)$ and $P\left(L|\tilde{l}\right) \le P(L|l)$. In other words, firm 1's information is more accurate than firm 2. In addition, we have $\frac{\partial P\left(H|\tilde{h}\right)}{\partial\zeta} > 0$ and $\frac{\partial P\left(L|\tilde{l}\right)}{\partial\zeta} > 0$. This means that an increase of ζ increases the accuracy of firm 2's information. With some algebra (more details in the Appendix A), we can show the following relationships.

Remark 1.

$$E\left[V|l\right] \le E\left[V|\tilde{l}\right] \le E\left[V\right] \le E\left[V|\tilde{h}\right] \le E\left[V|h\right]$$

The Remark suggests that the segmentation $\tilde{S}$ is less accurate than the segmentation S in indicating the value of a customer. Table 1 summarizes the notation of this model.

Before we present the model analysis, we first use a numerical example to illustrate the model setup. This numerical example will be used throughout the paper to explain the results. Suppose that $R=10$. The value of the prospective customer is therefore either $V=10$ or $V=0$. In other words, a valuable customer brings a revenue income

[4] For notational clarity, we use $Pr(.)$ to represent prior probabilities and $P(.)$ to represent posterior probabilities.

[5] In this paper, because the firms' segmentation information is derived from the same set of CDI data, it is reasonable to assume that firms' segmentation information is correlated.

Table 1
Notations of the model.

Notations	Description
$T=\{H,L\}$	The type of the perspective customer. $T=H$ ($T=L$) indicates a valuable (non-valuable) customer.
R	The revenue that a firm can generate from the valuable customer.
V	The value of the prospective customer. $V=R$ ($V=0$) for the valuable (non-valuable) customer.
λ	The prior probability that the prospective customer is valuable.
m_i, $i=\{1,2\}$	The promotional incentive that firm i offers to the prospective customer.
ψ	The probability that the CDI correctly classifies a valuable customer into the high segment for firrm 1.
ϕ	The probability that the CDI incorrectly classifies a non-valuable customer into the high segment for firm 1.
$S=\{h, l\}$	The segmentation for firm 1. $S=h$ ($S=l$) means the customer is classified into the high (low) segment for firm 1.
$\tilde{S}=\left\{\tilde{h},\tilde{l}\right\}$	The segmentation for firm 2. $\tilde{S}=\tilde{h}\left(\tilde{S}=\tilde{l}\right)$ means the customer is classified into the high (low) segment for firm 2.
$\Pr(h)$ (or $\Pr(l)$)	The probability that the CDI classifies the customer into the high (or low) segment for firm 1.
$\Pr\left(\tilde{h}\vert S\right)\left(or\Pr\left(\tilde{l}\vert S\right)\right)$	Given the segmentation S ($S=\{h, l\}$) for firm 1, the probability that the CDI classifies the customer into the high (or low) segment for firm 2.
$P\left(h\vert\tilde{S}\right)\left(or\,P\left(l\vert\tilde{S}\right)\right)$	Given the segmentation $\tilde{S}\left(\tilde{S}=\left\{\tilde{h},\tilde{l}\right\}\right)$ for firm 2, the posterior probability that the CDI classifies the customer into the high (or low) segment for firm 1.
ζ	The level of similarity between the segmentation services to these two firms. $\zeta=1$ means the services to these two firms are not differentiated; $\zeta=0$ means that the CDI serves only firm 1 exclusively.
$E[V\vert h]$ (or $E[V\vert l]$)	The expected value of the customer if the customer is classified into the high (or low) segment for firm 1.
$E\left[V\vert\tilde{h}\right]\left(or\,E\left[V\vert\tilde{l}\right]\right)$	The expected value of the customer if the customer is classified into the high (or low) segment for firm 2.
π_i	The expected profit of firm i in competition (exclusive of the payment to the CDI for the segmentation service).
Π	The profit of the CDI from selling services to firms.

10 to the firm over her lifetime relationship with the firm, whereas a non-valuable customer brings zero revenue. Suppose that $\psi=0.8$ and $\phi=0.2$. That is, if the customer is valuable (non-valuable), the CDI correctly classifies this customer into the high segment (low segment) for firm 1 with a probability 0.8. Therefore, suppose that $\lambda=0.6$ (i.e., the prior probability of valuable customer is 0.6), we have $Pr(h)=\lambda\psi+(1-\lambda)\phi=0.56$ (i.e., with a probability 0.56, the CDI classifies the customer into the high-segment for firm 1). If firm 1 observes that $S=h$ (i.e., a high-segment customer), it expects (using the Bayes' Rule) that the customer is a valuable customer with a posterior probability $\frac{\lambda\psi}{Pr(h)}=0.857>\lambda=0.6$. The expected value of the customer is $E[V|h]=8.57$. On the other hand, if firm 1 observes $S=l$ (i.e., a low-segment customer), it expects that the customer is a valuable customer with a posterior probability $1-\frac{(1-\lambda)(1-\phi)}{Pr(l)}=0.273<1-\lambda=0.4$. The expected value of the customer is $E[V|l]=2.73<E[V|h]$.

Suppose that the CDI provides a differential service $\tilde{S}$ to firm 2 with a similarity factor $\zeta=0.5$. Then we have $Pr\left(\tilde{h}|h\right)=Pr\left(\tilde{l}|l\right)=\frac{1+\zeta}{2}=0.75$. This means that if the CDI classifies the customer into the high segment (low segment) for firm 1, it also classifies this customer into the high segment (low segment) for firm 2 with a probability 0.75. Therefore, according to Eq. (6), when firm 2 observes $\tilde{S}=\tilde{h}$ (i.e., a high-segment customer), it expects that the customer is also a high-segment customer for firm 1 with a posterior probability $P\left(h|\tilde{h}\right)=0.79$. Firm 2 thus expects the value of the customer to be $E\left[V|\tilde{h}\right]=7.36$. When firm 2 observes $\tilde{S}=\tilde{l}$ (i.e., a low-segment customer), it expects that the customer is also a low-segment customer for firm 1 with a posterior probability $P\left(l|\tilde{l}\right)=0.70$. Firm 2 thus expects the value of the customer to be $E\left[V|\tilde{l}\right]=4.47<E\left[V|\tilde{h}\right]$. Note that we have $E[V|l]<E\left[V|\tilde{l}\right]<E[V]=6<E\left[V|\tilde{h}\right]<E[V|h]$, which means that the segmentation S for firm 1 is more accurate in revealing the customer value than the segmentation $\tilde{S}$ for firm 2.

The timing of events is as follows. First, the CDI designs the segmentation S for firm 1 by determining ψ and ϕ. In addition to firm 1, the CDI decides whether or not to also serve firm 2. If the CDI also serves firm 2, it determines ζ to differentiate the segmentation $\tilde{S}$ for firm 2 from the segmentation S for firm 1. The CDI also determines how much to charge each firm. After determining all these details, the CDI makes take-it-or-leave-it offers to firms. Second, firms determine whether to accept the CDI's offers and use the CDI's services. Firms who use the CDI's services observe the customer segmentation (firm 1 observes the customer segmentation S and firm 2, if served, observes the customer segmentation $\tilde{S}$). Third, firms simultaneously decide their promotional incentives sent to the prospective customer, m_i, where $i=1,2$. Finally, the customer patronizes the firm which offers the higher promotional incentive. The value of the customer is realized for the winning firm. We assume that everything is common knowledge except the customer's value V and the firms' segmentations S and $\tilde{S}$. In practice, CDIs such as Catalina Marketing often keep their service specifications transparent (e.g., the length of transaction history used in data analysis, redemption rate of target marketing services). Therefore, we assume that firms can assess the service quality and differentiation (i.e., ψ, ϕ, and ζ).

3. Firm competition

Following backward induction, we first consider the firms' competition given the customer segmentations, and then consider the CDI's strategies based on the equilibrium of firm competition. In this section, we derive the Bayesian Nash Equilibrium of the firms' competition. In the competition, firm i chooses its promotional incentive, m_i, to maximize its expected profit, denoted by π_i, $i=1,2$.

When $\zeta=1$, firms' segmentations are exactly the same. Therefore, firms engage in a Bertrand competition. Firms both offer $m=E[V|l]$ when $S=\tilde{S}=l$ and $m=E[V|h]$ when $S=\tilde{S}=h$. We next focus on the equilibria when $0\le\zeta<1$. We find that no pure-strategy equilibrium exists for the firms' competition. Suppose that firm 2 offers a deterministic promotional incentive m_2. Firm 1 always offers a promotional incentive higher than m_2 if $E[V|S]>m_2$. As a result, firm 2 can only acquire the customer when $E[V|S]\le m_2$ and make a non-positive profit. This is a typical case of *Winner's Curse*. The only equilibrium is a mixed-strategy equilibrium. In marketing, mixed strategies can be interpreted as the frequent dispersion of firms' sales offers or promotions [18]. The proof of no pure-strategy equilibrium is in the Appendix A.

Let $G_1(m|S)=Pr(m_1\le m|S)$ denote the equilibrium cumulative distribution function (CDF) of firm 1's promotional incentive conditional on its customer segmentation S. Let $G_2\left(m|\tilde{S}\right) = Pr\left(m_2 \le m|\tilde{S}\right)$ denote the equilibrium CDF of firm 2's promotional incentive conditional on its customer segmentation $\tilde{S}$. Firm 1's expected profit when offering m_1, conditional on S, can be represented as

$$\pi_1(m_1|S) = Pr\left(\tilde{h}|S\right)G_2\left(m_1|\tilde{h}\right)\left(E\left[V|S\right]-m_1\right) + Pr\left(\tilde{l}|S\right)G_2\left(m_1|\tilde{l}\right)\left(E\left[V|S\right]-m_1\right). \quad (7)$$

The first term in the right-hand side (RHS) of Eq. (7) is firm 1's expected profit when the customer is classified into the high segment for firm 2 (i.e., $\tilde{S} = \tilde{h}$). The second term is firm 1's expected profit when the customer is classified into the low segment for firm 2 (i.e., $\tilde{S} = \tilde{l}$). Similarly, firm 2's expected profit when offering m_2, conditional on $\tilde{S}$, can be represented as

$$\pi_2\left(m_2|\tilde{S}\right) = P\left(h|\tilde{S}\right)G_1\left(m_2|h\right)\left(E\left[V|h\right]-m_2\right) + P\left(l|\tilde{S}\right)G_1\left(m_2|l\right)\left(E\left[V|l\right]-m_2\right). \quad (8)$$

The first term in the RHS of Eq. (8) is firm 2's expected profit when the customer is classified into the high segment for firm 1 (i.e., $S = h$). The second term is firm 2's expected profit when the customer is classified into the low segment for firm 1 (i.e., $S = l$). Proposition 1 characterizes the equilibrium distributions of firms' promotional incentives when $Pr(h) \le Pr(l)$ (i.e., for firm 1, the customer is more likely to be classified into the low segment than into the high segment).

Proposition 1. When $Pr(h) \le Pr(l)$ and $0 \le \zeta < 1$,

1. If the customer is classified into the low segment for firm 1 (i.e., $S = l$), firm 1 offers a promotional incentive $m_1 = E[V|l]$;
2. If the customer is classified into the low segment for firm 2 (i.e., $\tilde{S} = \tilde{l}$), firm 2 offers a promotional incentive $m_2 = E[V|l]$;
3. If the customer is classified into the high segment for firm 1 (i.e., $S = h$), firm 1 offers a randomized promotional incentive with the CDF

$$G_1(m|h) = \frac{P\left(l|\tilde{h}\right)\left(m-E\left[V|l\right]\right)}{P\left(h|\tilde{h}\right)\left(E\left[V|h\right]-m\right)}, m \in \left[E[V|l], E\left[V|\tilde{h}\right]\right].$$

4. If the customer is classified into the high segment for firm 2 (i.e., $\tilde{S} = \tilde{h}$), firm 2 offers a randomized promotional incentive with the CDF

$$G_2\left(m|\tilde{h}\right) = \frac{E\left[V|h\right]-E\left[V|\tilde{h}\right]-Pr\left(\tilde{l}|h\right)\left(E\left[V|h\right]-m\right)}{Pr\left(\tilde{h}|h\right)\left(E\left[V|h\right]-m\right)}, m \in \left[E\left[V|l\right], E\left[V|\tilde{h}\right]\right].$$

To help understand how firms compete given the customer segmentations, we use a numerical example to illustrate Proposition 1. In this example, we use the same specifications as in the numerical example in the previous section. Specifically, $R = 10$, $\psi = 0.8$, $\phi = 0.2$, and $\zeta = 0.5$. Therefore, we have $Pr\left(\tilde{h}|h\right) = Pr\left(\tilde{l}|l\right) = 0.75$. We let $\lambda = 0.4$ (i.e., the prior probability of valuable customer is 0.4). We therefore have $Pr(h) = 0.44$, i.e., with a probability 0.44, the customer is classified into the high-segment for firm 1. Note that $Pr(h) < Pr(l) = 0.56$. With this $Pr(h)$, we can derive the following expected values

$$E\left[V|h\right] = 7.27; E\left[V|l\right] = 1.43; E\left[V|\tilde{h}\right] = 5.54; E\left[V|\tilde{l}\right] = 2.62,$$

and CDFs

$$G_1(m|h) = \frac{0.42(m-1.43)}{7.27-m}; G_2\left(m|\tilde{h}\right) = \frac{2.30-0.33(7.27-m)}{7.27-m}, m \in [1.43, 5.54].$$

Fig. 1(a) depicts the supports of firms' promotional incentive distributions when $Pr(h) \le Pr(l)$. As Proposition 1 shows, when $S = l$, firm 1 expects that the customer value is $E[V|l] = 1.43$. Firm 1 is thus unwilling to offer any promotional incentive higher than 1.43. Because the lowest expected value of the customer across firms 1 and 2 is also 1.43 and both firms compete for the customer, firm 1's promotional incentive for the customer is always $m_1 = 1.43$ and firm 1's expected profit is zero. When $S = h$, firm 1's expected value for the consumer is $E[V|h] = 7.27$, which is higher than $E\left[V|\tilde{h}\right]$ and $E\left[V|\tilde{l}\right]$. Therefore, firm 1 is more willing to win this customer than firm 2. In equilibrium, firm 1 randomizes its promotional incentive in such a way that firm 2 earns a zero expected profit.

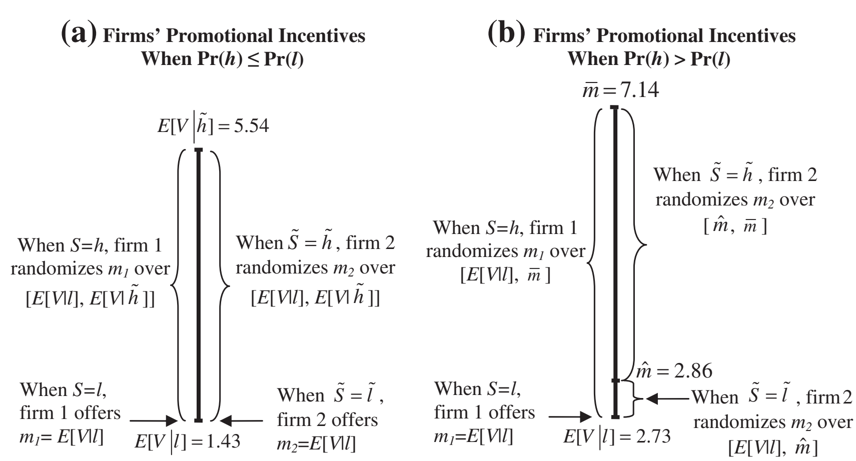

Fig. 1. The equilibrium of firm competition.

Now let us consider firm 2's strategies. When $\tilde{S} = \tilde{l}$, firm 2 expects that the customer value is $E\left[V|\tilde{l}\right] = 2.62 > E[V|l]$. We find that firm 2's equilibrium strategy is also to offer a deterministic promotional incentive $m_2 = E[V|l] = 1.43$. Firm 2 gives up the competition because of its information disadvantage. When the customer is classified into the low segment for firm 1 ($S = l$), firm 2 can win for sure if it offers $m_2 > 1.43$. However, it is very likely that firm 2 acquires a non-valuable customer and its expected profit is negative. In this case, firm 2 may suffer from *the Winner's Curse*. On the other hand, when the customer is classified into the high segment for firm 1 ($S = h$), firm 1 will compete aggressively for this customer and randomize its promotional incentive in a way that firm 2 cannot make a positive expected profit. Therefore, when $\tilde{S} = \tilde{l}$, firm 2 cannot do better than offering $m_2 = E[V|l] = 1.43$ (and making zero expected profit).

When $\tilde{S} = \tilde{h}$, firm 2 randomizes its promotional incentive. The upper bound of the firms' promotional incentive distributions is 5.54 because $E\left[V|\tilde{h}\right] = 5.54$ is the highest expected value for firm 2. In the mixed-strategy equilibrium, firm 1 and firm 2 randomize their promotional incentive over the same support [1.43,5.54]. Fig. 2 depicts the CDFs of firms' promotional incentives $G_1(m|h)$ and $G_2\left(m|\tilde{h}\right)$. Note that $G_2\left(m|\tilde{h}\right)$ has a mass point at the lower bound $m = 1.43$. This means that firm 2 provides $m_2 = 1.43$ with a positive probability of 0.06.

Next, we consider the case when $Pr(h) > Pr(l)$ (i.e., for firm 1, the customer is more likely to be classified into the high segment than into the low segment). Eq. (2) indicates that $Pr(h) > Pr(l)$ occurs if and only if $2\lambda > \frac{1-2\phi}{\psi-\phi}$. Also, when $\phi > 0.5$, we always have $Pr(h) > Pr(l)$. This means that we have $Pr(h) > Pr(l)$ when ϕ is sufficiently large. Proposition 2 characterizes the equilibrium distributions of firms' promotional incentives when $Pr(h) > Pr(l)$.

Proposition 2. When $Pr(h) > Pr(l)$ and $0 \le \zeta < 1$,

1. If the customer is classified into the low segment for firm 1 (i.e., $S = l$), firm 1 offers a promotional incentive $m_1 = E[V|l]$;
2. If the customer is classified into the low segment for firm 2 (i.e., $\tilde{S} = \tilde{l}$), firm 2 offers a randomized promotional incentive with the CDF

$$G_2\left(m|\tilde{l}\right) = \frac{E\left[V|h\right] - \overline{m}}{Pr\left(\tilde{l}|h\right)\left(E\left[V|h\right] - m\right)}, m \in [E[V|l], \hat{m}].$$

3. If the customer is classified into the high segment for firm 1 (i.e., $S = h$), firm 1 offers a randomized promotional incentive with the CDF

$$G_1(m|h) = \begin{cases} \dfrac{P\left(l|\tilde{l}\right)\left(m - E[V|l]\right)}{P\left(h|\tilde{l}\right)\left(E\left[V|h\right] - m\right)}, m \in [E[V|l], \hat{m}]; \\ \dfrac{E\left[V|\tilde{h}\right] - \overline{m} - P\left(l|\tilde{h}\right)\left(E\left[V|l\right] - m\right)}{P\left(h|\tilde{h}\right)\left(E\left[V|h\right] - m\right)}, m \in [\hat{m}, \overline{m}]. \end{cases}$$

4. If the customer is classified into the high segment for firm 2 (i.e., $\tilde{S} = \tilde{h}$), firm 2 offers a randomized promotional incentive with the CDF

$$G_2\left(m|\tilde{h}\right) = \frac{E\left[V|h\right] - \overline{m} - Pr\left(\tilde{l}|h\right)\left(E\left[V|h\right] - m\right)}{Pr\left(\tilde{h}|h\right)\left(E\left[V|h\right] - m\right)}, m \in [\hat{m}, \overline{m}].$$

where the cutoff levels, $\hat{m}$ and $\overline{m}$, are given in the Appendix A.

To explain Proposition 2, we use a numerical example again. We use the same specifications as in the numerical example for Proposition 1. Specifically, $R = 10$, $\psi = 0.8$, $\phi = 0.2$, and $\zeta = 0.5$. We let $\lambda = 0.6$ (i.e., the prior probability of valuable customer is 0.6). We therefore have $Pr(h) = 0.56 > Pr(l) = 0.44$. With this $Pr(h)$, we can derive the following values

$$E\left[V|h\right] = 8.57; E\left[V|l\right] = 2.73; E\left[V|\tilde{h}\right] = 7.36; E\left[V|\tilde{l}\right] = 4.47;$$
$$\hat{m} = 2.86; \overline{m} = 7.14,$$

and CDFs

$$G_2\left(m|\tilde{l}\right) = \frac{5.71}{8.57 - m}, m \in [2.73, 2.86].$$

$$G_2\left(m|\tilde{h}\right) = \frac{0.33m - 0.94}{8.57 - m}, m \in [2.86, 7.14].$$

$$G_1\left(m|h\right) = \begin{cases} \dfrac{2.36(m - 2.73)}{8.57 - m}, m \in [2.73, 2.86]; \\ \dfrac{0.26m - 0.44}{8.57 - m}, m \in [2.86, 7.14]. \end{cases}$$

Fig. 1(b) depicts the supports of firms' promotional incentive distributions when $Pr(h) > Pr(l)$. As Proposition 2 shows, when $S = l$, firm 1 expects that the customer value is $E[V|l] = 2.73$ and therefore offers $m_1 = 2.73$. When $S = h$, firm 1 competes less aggressively than it does in the case where $Pr(h) \le Pr(l)$. In particular, the upper bound of the firms' promotional incentives is $\overline{m} = 7.14$, which is lower than $E\left[V|\tilde{h}\right] = 7.36$ (i.e., the upper bound when $Pr(h) \le Pr(l)$).

In Proposition 2, firm 2 also randomizes its promotional incentive for its low-segment customer. When $\tilde{S} = \tilde{l}$, firm 2 no longer finds it optimal to offer a promotional incentive $m_2 = E[V|l] = 2.73$. Instead, firm 2 competes with firm 1 by randomizing its promotional incentive over the support $[E[V|l], \hat{m}] = [2.73, 2.86]$. However, in equilibrium, firm 2 still earns a zero expected profit in competing for its low-segment customer. This is because firm 1 has information advantage and randomizes its promotional incentive in such a way that firm 2 cannot gain a positive expected profit from its low-segment customer. Fig. 3 shows the CDF of firm 2's promotional incentive when $\tilde{S} = \tilde{l}$, i.e., $G_2\left(m|\tilde{l}\right)$. Note that $G_2\left(m|\tilde{l}\right)$ has a mass point at $m_2 = E[V|l] = 2.73$. This means that when $\tilde{S} = \tilde{l}$, firm 2 provides $m_2 = 2.73$ with a positive probability of 0.977.

Proposition 2 indicates that when firm 1 randomizes its promotional incentive to its high-segment customers (i.e., $S = h$), the

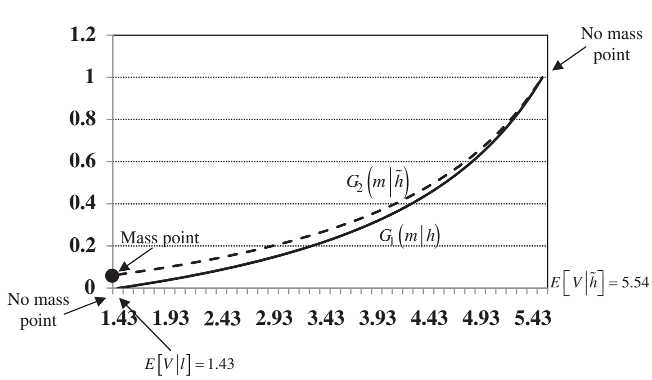

Fig. 2. Examples of the cumulative distribution functions of firms' promotional incentives when $\Pr(h) \le \Pr(l)$.

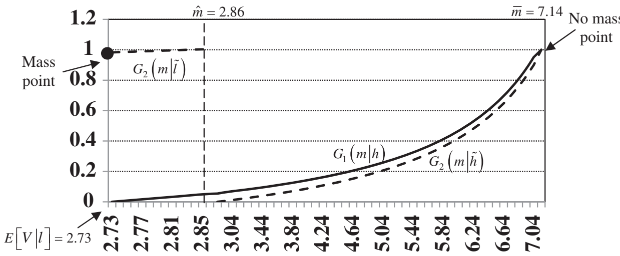

Fig. 3. Examples of the cumulative distribution functions of firms' promotional incentives when $\Pr(h) > \Pr(l)$.

equilibrium distribution function $G_1(m|h)$ is kinked at $\hat{m} = 2.86$. This is caused by the difference between the customer segmentation for firm 1 and that for firm 2. The high-segment customer for firm 1 may be classified into either the high segment (i.e., $\tilde{S} = \tilde{h}$) or low segment (i.e., $\tilde{S} = \tilde{l}$) for firm 2. Firm 2 randomizes its promotional incentives over $[E[V|l], \hat{m}]$ when $\tilde{S} = \tilde{l}$ and over $[\hat{m}, \overline{m}]$ when $\tilde{S} = \tilde{h}$. Firm 1's promotional incentives targeting its high segment have to compete with these two types of firm 2's promotional incentives. Therefore, $G_1(m|h)$ is kinked at $\hat{m}$. Fig. 3 shows the CDF of firm 1's promotional incentive when $S = h$, i.e., $G_1(m|h)$, and the CDFs of firm 2's promotional incentive, $G_2\left(m|\tilde{h}\right)$ and $G_2\left(m|\tilde{l}\right)$.

Next, we consider the firms' expected profits in competition. We use π_1 and π_2 to denote the expected profits of firm 1 and firm 2, respectively. Proposition 3 summarizes the equilibrium profits of firms in competition.

Proposition 3. When $0 \leq \zeta < 1$, in equilibrium,

1. If $Pr(h) \leq Pr(l)$, firm 1 makes a positive expected profit $\pi_1 = Pr(h)\left(E[V|h] - E\left[V|\tilde{h}\right]\right)$, and firm 2 makes zero expected profit;
2. If $Pr(h) > Pr(l)$, firm 1 makes a positive expected profit $\pi_1 = Pr(h)(E[V|h] - \overline{m})$. Firm 2 makes a positive expected profit $\pi_2 = Pr\left(\tilde{h}\right)\left(E\left[V|\tilde{h}\right] - \overline{m}\right)$ when $\zeta > 0$, and a zero expected profit when $\zeta = 0$;
3. Firm 1's expected profit is always higher than firm 2's expected profit.

Proposition 3 illustrates and compares firms' expected profits. When $Pr(h) \leq Pr(l)$, only firm 1 earns a positive expected profit in competition. We ignore the trivial case of $\pi_1 = 0$ when $Pr(h) = 0$ (i.e., the customer is never classified into the high segment for firm 1). Note that compared to firm 2, firm 1 has more accurate customer segmentation, and thus better information about the customer. Therefore, it earns a positive information rent in competition. Such finding is consistent with the existing literature on common-value auctions with asymmetrically informed bidders [6].

When $Pr(h) > Pr(l)$, however, Proposition 3.2 illustrates that both firms earn positive expected profits in competition when $0 < \zeta < 1$. In other words, both firms reap information rents when the CDI serves both of them but provides differentiated services. Note that even though the customer segmentation for firm 2 is not as accurate as that for firm 1, firm 1 does not directly observe the segmentation outcome for firm 2. Thus firm 2's information about the customer is also private information although this information is not as good as firm 1's private information. Firm 2 can therefore earn information rent from this private information. Proposition 3.3 shows that firm 1's information rent is always higher than that of firm 2. This result is intuitive since firm 1 has better customer information than firm 2.

Proposition 3 presents an important insight on the competition between two firms with asymmetric information. Firm 2, even though less informed, also has private information. However, whether firm 2 can reap information rent from this private information is dependent on firm 1's information, i.e., whether the condition $Pr(h) > Pr(l)$ holds. When $Pr(h) \leq Pr(l)$, firm 1 competes aggressively and strives to win the customer when $S = h$, regardless of whether it is a high-segment or low-segment customer for firm 2. In the mixed-strategy equilibrium, firm 1 randomizes its promotional incentives in such a way that firm 2 cannot make any positive expected profit in the competition when $\tilde{S} = \tilde{h}$ or $\tilde{S} = \tilde{l}$. However, when $Pr(h) > Pr(l)$, firm 1 competes less aggressively and focuses on winning firm 2's low-segment customer. In other words, since firm 1 identifies this customer as a valuable customer most of the time, firm 1 would like to take the chance that firm 2 may not identify this customer in the same way as firm 1. By doing so, firm 1 saves its expenditure on promotional incentive although it will more likely lose the competition. In the mixed-strategy equilibrium, firm 1 randomizes its promotional incentives in such a way that firm 2 cannot make positive expected profit in the competition when $\tilde{S} = \tilde{l}$. However, if the customer happens to be in firm 2's high-segment ($\tilde{S} = \tilde{h}$), the less aggressiveness of firm 1 enables firm 2 to make a positive expected profit $E\left[V|\tilde{h}\right] - \overline{m}$ when $0 < \zeta < 1$. That is why firm 2's overall expected profit is $\pi_2 = Pr\left(\tilde{h}\right)\left(E\left[V|\tilde{h}\right] - \overline{m}\right)$.

Proposition 3 captures the case where $0 \leq \zeta < 1$. If $\zeta = 1$, the customer segmentations for firm 1 and for firm 2 are the same and there is no service differentiation. In this case, firms have the same information about the value of the customer. They engage in a head-to-head competition and neither makes a positive profit (i.e., $\pi_1 = \pi_2 = 0$). In this regard, the service differentiation helps firms avoid the destructive head-to-head competition.

We next examine how service differentiation influences the firm profits in competition. The degree of service differentiation is captured by ζ. When ζ increases, the customer segmentations for the two firms become more similar. Thus, the increase of ζ reduces the service differentiation between firms. Proposition 4 characterizes how the change in ζ influences the firms' expected profits.

Proposition 4.

1. When $Pr(h) \leq Pr(l)$, firm 1's expected profit is always decreasing in ζ; firm 2's expected profit is always zero;
2. When $Pr(h) > Pr(l)$, firm 1's expected profit is increasing in ζ when $\zeta \in [0, \max\{0, \zeta_1\}]$ and decreasing in ζ when $\zeta \in [\max\{0, \zeta_1\}, 1]$, firm 2's expected profit is increasing in ζ when $\zeta \in [0, \zeta_2]$ and decreasing in ζ when $\zeta \in [\zeta_2, 1]$. The cutoff levels $\zeta_1 = \frac{\gamma - 2\sqrt{(2\gamma-1)(1-\gamma)}}{3\gamma - 2} < \zeta_2 = \frac{1 - 2\sqrt{(1-\gamma)}}{4\gamma - 3}$, where $\gamma = Pr(h)$.

When $Pr(h) \leq Pr(l)$, only firm 1 makes a positive expected profit in competition (see Proposition 3). When ζ increases, firm 2's information is more similar to firm 1's information. A higher ζ leads firm 2 to be more confident that its high-segment customer is more likely to be a high-segment customer for firm 1. Therefore, firm 2 is more willing to compete for its high-segment customer. Consequently, firms compete more head-to-head and firm 1's expected profit decreases.

When $Pr(h) > Pr(l)$, both firms make positive expected profits in competition when $\zeta > 0$. Again, the increase in ζ results in less service differentiation and more similar information for firms. However, Proposition 4.2 shows that firms' profits are not monotonically decreasing in ζ. This is because the change in ζ generates two countervailing effects on firm 2's competitive aggressiveness. In addition to the effect we discussed in the above paragraph, a higher ζ also generates another effect—when ζ increases, firm 2 is also more confident that its low-segment customer is more likely to be a low-segment customer for firm 1. Therefore, firm 2 is less willing to compete for its low-segment customer. In the case of $Pr(h) \leq Pr(l)$, only the first effect exists.

Therefore firms' profits are never increasing in ζ. In the case of $Pr(h)>Pr(l)$, both effects exist. Whether or not firm 2 becomes more aggressive in general depends on the tradeoff between these two countervailing forces. When ζ is small enough, the likelihood that the customer is classified into the low segment for firm 2 is relatively high and thus the first force is dominant. As a result, the increase in ζ makes firm 2 less aggressive in general. The firms' competition is softened. When ζ is large enough, the likelihood that the customer is classified into the high segment for firm 2 is relatively high and thus the second force is dominant. As a result, the increase in ζ makes firm 2 more aggressive in general. The firms' competition is intensified.

We next illustrate the impact of information similarity ζ on the competition intensity using firms' expected promotional incentives. When firms compete more (less) aggressively, they offer higher (lower) promotional incentives. The firms' expected promotional incentives, $E[m_1]$ and $E[m_2]$, can be used to capture the firms' competitive aggressiveness. $E[m_1]$ and $E[m_2]$ are the average levels of firms' promotional incentives, weighted by the probability distributions of mixed-strategies ($G_1(m|S)$ and $G_2\left(m|\tilde{S}\right)$) and the probabilities of segmentation outcomes ($Pr(S)$ and $Pr\left(\tilde{S}\right)$). Firms 1 and 2's overall expected promotional incentives are respectively

$$E\left[m_1\right]=Pr(h)E\left[m_1|h\right]+Pr(l)E\left[m_1|l\right],$$
$$E\left[m_2\right]=Pr\left(\tilde{h}\right)E\left[m_2|\tilde{h}\right]+Pr\left(\tilde{l}\right)E\left[m_2|\tilde{l}\right].$$

When $Pr(h)\leq Pr(l)$, firms 1 and 2's expected promotional incentives given a signal are respectively

$$E[m_1|h]=\int_{E[V|l]}^{E[V|\tilde{h}]}mdG_1(m|h) \text{ and } E[m_1|l]=E[V|l],$$

$$E\left[m_2|\tilde{h}\right]=\int_{E[V|l]}^{E[V|\tilde{h}]}mdG_2\left(m|\tilde{h}\right)+G_2\left(m|\tilde{h}\right)\Big|_{m=E[V|l]}E\left[V|l\right] \text{ and } E\left[m_2|\tilde{l}\right]=E\left[V|l\right].$$

When $Pr(h)>Pr(l)$, firms 1 and 2's expected promotional incentives given a signal are respectively

$$E[m_1|h]=\int_{E[V|l]}^{\hat{m}}mdG_1(m|h)+\int_{\hat{m}}^{\overline{m}}mdG_1(m|h) \text{ and } E[m_1|l]=E[V|l],$$

$$E\left[m_2|\tilde{h}\right]=\int_{\hat{m}}^{\overline{m}}mdG_2\left(m|\tilde{h}\right) \text{ and } E\left[m_2|\tilde{l}\right]=\int_{E[V|l]}^{\hat{m}}mdG_2\left(m|\tilde{l}\right)+G_2\left(m|\tilde{l}\right)\Big|_{m=E[V|l]}E\left[V|l\right].$$

Fig. 4 illustrates how $E[m_1]$ and $E[m_2]$ change with ζ. The parameter specifications used in Fig. 4 are consistent with those in the numerical examples for Propositions 1 and 2. Specifically, $R=10$, $\psi=0.8$, and $\phi=0.2$. We use $\lambda=0.4$ for Fig. 4(a) and $\lambda=0.8$ for Fig. 4(b) to better illustrate the effect. Fig. 4(a) shows that both $E[m_1]$ and $E[m_2]$ are always increasing in ζ when $Pr(h)\leq Pr(l)$. Firms compete more aggressively when the level of information asymmetry becomes lower. Fig. 4(b) shows that both $E[m_1]$ and $E[m_2]$ are first decreasing and then increasing in ζ when $Pr(h)>Pr(l)$. This indicates when ζ is small, the increase of ζ makes both firms less aggressive in offering promotional incentives. In other words, the increase of ζ softens the competition. This explains why both firms' expected profits are increasing in ζ when ζ is small. When ζ is large, the increase of ζ makes both firms more aggressive in offering promotional incentives. In other words, the increase of ζ intensifies the competition. This explains why both firms' expected profits are decreasing in ζ when ζ is large.

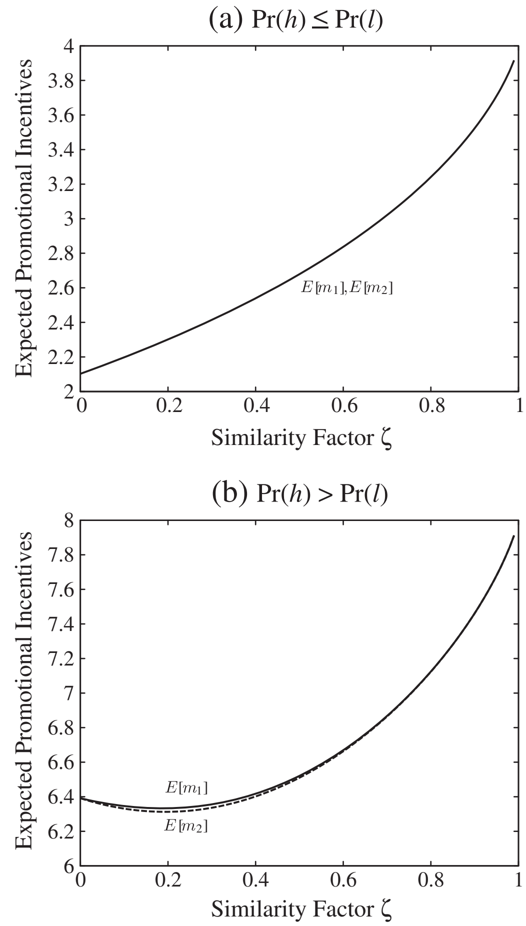

Fig. 4. Firms' expected promotional incentives as a funtion of ζ.

4. CDI service design

In this section, we consider the CDI's strategies in providing services of customer segmentation to firms. Given the competition equilibrium characterized in the previous sections, we consider the CDI's decision on the optimal level of similarity factor ζ. If $\zeta=0$, $\tilde{S}$ does not provide additional information to firm 2 and firm 2 only knows the prior probability of valuable customer. It is equivalent to the case that the CDI serves firm 1 exclusively. If $0<\zeta<1$, firms have different information about the customer. It can be considered as the case that the CDI serves both firms but provides differentiated services. If $\zeta=1$, the CDI serves both firms and there is no service differentiation. As a result, firms have the same information about the customer.

If the CDI only serves one firm, it can make a take-it-or-leave-it offer to firm 1 at a price π_1 and let firm 1 decide whether or not to accept it. If firm 1 does not accept it, the CDI makes the same offer to firm 2. In equilibrium, firm 1 accepts the offer. The logic is as follows. As Proposition 3 shows, the firm with private information makes a positive profit and the firm without private information makes zero profit. If firm 1 rejects the offer but firm 2 accepts it, firm 1 becomes the firm without private information and will earn zero profit in competition. If neither firm accepts the offer and hence neither firm has private information, the competition becomes Bertrand competition and both firms make a zero profit. Therefore, firm 1 is willing to accept the offer. In equilibrium, the CDI earns a profit $\Pi=\pi_1$. In other words, the CDI appropriates all the surplus.

If the CDI serves both firms, in line with the existing literature (e.g., [4,20]), the CDI can make take-it-or-leave-it offers sequentially to the two firms. The CDI offers firm 1 a price π_1 and then offers firm 2 a price π_2. If one firm rejects the offer and the other firm accepts the offer, the rejecting firm becomes the firm without private information and makes a zero profit in competition. If neither firm accepts the offer, again, the competition becomes Bertrand competition and both firms make a zero profit. Therefore, both firms are willing to accept the offer. In equilibrium, the CDI's profit is

$$\Pi = \pi_1 + \pi_2. \tag{9}$$

The CDI again appropriates all the surplus. Proposition 5 illustrates the impact of the similarity factor ζ on the CDI's profit.

Proposition 5.

1. When $Pr(h) \le Pr(l)$, the CDI's expected profit is always decreasing in ζ.
2. When $Pr(h) > Pr(l)$, the CDI's expected profit is increasing in ζ when $\zeta \in [0, \max\{0, \zeta_0\}]$ and decreasing in ζ when $\zeta \in [\max\{0, \zeta_0\}, 1]$ where $\zeta_0 = \frac{-1+2\gamma+\gamma^2-2\sqrt{(2\gamma-1)(1-\gamma)(4\gamma^2+\gamma-1)}}{11\gamma^2-12\gamma+3}$ and $\gamma = Pr(h)$. Moreover, ζ_0 satisfies that $\zeta_1 < \zeta_0 < \zeta_2$.

When $Pr(h) \le Pr(l)$, Proposition 3 shows that only firm 1 earns a positive expected profit in competition. Therefore, using a take-it-or-leave-it offer, the CDI can appropriate firm 1's surplus and earn $\Pi = \pi_1$. The CDI's expected profit is always decreasing in ζ because as Proposition 4 shows, firm 1's expected profit (excluding the service fee) is decreasing in ζ. As a result, the CDI will choose $\zeta = 0$, i.e., the CDI serves firm 1 exclusively and thus maximizes the information difference between firms.

When $Pr(h) > Pr(l)$, firms may both earn positive expected profits in the competition. By making take-it-or-leave-it offers to both firms, the CDI can appropriate both firms' surplus and the CDI's profit is $\Pi = \pi_1 + \pi_2$. As Proposition 4 shows, when ζ is small enough, the increases of ζ may soften the firms' competition. When ζ is large enough, the increase of ζ intensifies the firms' competition. As a result, the CDI may benefit by serving both firms and maintaining an intermediate level of service differentiation. Proposition 5 indicates that when $\zeta_0 > 0$, the CDI should serve both firms and differentiate the services by choosing the similarity factor ζ_0.

It is worth remarking that $\frac{d\zeta_0}{d\gamma} > 0$, i.e., the optimal level of similarity is increasing in $Pr(h)$ (note that $\gamma = Pr(h)$). In other words, when it is more likely that the customer is classified into the high segment for firm 1, the CDI makes the two firms' services more similar. The rationale is that, on average, both firms profit from the high-segment customer but not from the low-segment customer in the competition. When $Pr(h)$ increases, by making firm 2's customer segmentation more similar to that of firm 1, the CDI can also make the customer more likely to be classified as the high-segment customer for firm 2. In this way, the CDI improves firm 2's profitability and eventually appropriates more surplus from firm 2.

Proposition 6 characterizes how the CDI's service differentiation strategies are based on $Pr(h)$.

Proposition 6. When $Pr(h) \le \frac{\sqrt{3}}{3}$, the CDI only serves firm 1. When $Pr(h) > \frac{\sqrt{3}}{3}$, the CDI serves both firms and chooses a similarity factor ζ_0 (as defined in Proposition 5) to differentiate the services to firms.

From Proposition 5, the optimal ζ for the CDI is zero when $Pr(h) \le \frac{1}{2}$, and is $\max\{0, \zeta_0\}$ when $Pr(h) > \frac{1}{2}$. The condition $\zeta_0 > 0$ requires that $Pr(h) > \frac{\sqrt{3}}{3}$. Therefore, when $0 \le Pr(h) \le \frac{\sqrt{3}}{3}$, the CDI chooses $\zeta = 0$, i.e., only offering an exclusive service to firm 1. When $Pr(h) > \frac{\sqrt{3}}{3}$, the CDI chooses $\zeta = \zeta_0 > 0$, i.e., the CDI serves both firms and differentiates the services to firms.

We use a numerical example to illustrate the CDI's service differentiation. Consistent with the previous numerical examples, we assume that $R = 10$, $\psi = 0.8$, and $\phi = 0.2$. Fig. 5 shows how the CDI's optimal ζ changes with $Pr(h)$. The CDI chooses $\zeta = 0$ (i.e., serving only firm 1) when $Pr(h) \le \frac{\sqrt{3}}{3}$. When $Pr(h) > \frac{\sqrt{3}}{3}$, the optimal ζ_0 is increasing in $Pr(h)$.

Next we consider how the CDI maximizes its profit by controlling the information accuracy of its services. Based on Proposition 3 and Eq. (9), the CDI's profit can be represented as

$$\Pi = \pi_1 + \pi_2 = \begin{cases} Pr(h)\left(E\left[V|h\right] - E\left[V|\tilde{h}\right]\right), & \text{when } Pr(h) \le \frac{1}{2} \\ Pr(h)\left(E\left[V|h\right] - \overline{m}\right) + Pr\left(\tilde{h}\right)\left(E\left[V|\tilde{h}\right] - \overline{m}\right), & \text{when } Pr(h) > \frac{1}{2}. \end{cases} \tag{10}$$

The CDI chooses ψ, ϕ, and ζ to maximize its profit. By adjusting ψ and ϕ, the CDI controls $Pr(h)$ (i.e., the probability that the prospective customer is classified into the high segment for firm 1) because $Pr(h) = \lambda\psi + (1-\lambda)\phi$ (please see Eq. (2)). Therefore, when ψ and ϕ are determined, the values of $Pr(h)$ and $E[V|h]$ in Eq. (10) are determined. By adjusting ζ, the CDI controls to what extent the segmentation for firm 2 is different from the segmentation for firm 1 (please see Eq. (4)). When ζ is determined in addition to ψ and ϕ, the values of $E\left[V|\tilde{h}\right]$, $Pr\left(\tilde{h}\right)$ and $\overline{m}$ in Eq. (10) are also determined. As Propositions 5 and 6 indicate, the optimal ζ for the CDI is essentially dependent on $Pr(h)$. Therefore, when ψ and ϕ are determined, the optimal ζ is also determined. The CDI's decision variables are ψ and ϕ. The CDI's problem is

$$\begin{aligned} &\max_{\{\psi,\phi\}} \Pi \\ &\text{s.t.} 0 \le \phi < \psi \le 1. \end{aligned} \tag{11}$$

Proposition 7 shows how the CDI's service design depends on the market composition λ.

Proposition 7.

1. When $0 < \lambda \le \frac{\sqrt{3}}{3}$, the CDI chooses $\psi = 1$, $\phi = 0$ and $\zeta = 0$;
2. When $\frac{\sqrt{3}}{3} < \lambda \le 1$, the CDI chooses $\psi = 1$, $\phi = 0$ and $\zeta = \zeta_0$ in differentiating the services to firms.

The CDI maximizes its profit by choosing $\psi = 1$ and $\phi = 0$. This result is independent of the market composition λ. This implication

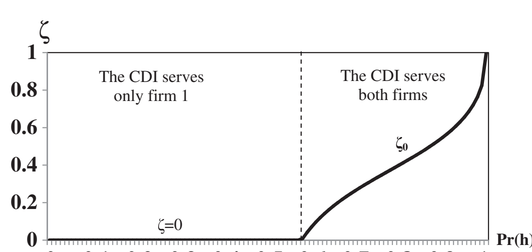

Fig. 5. The optimal similarity factor ζ_0 of firm 2's customer segmentation.

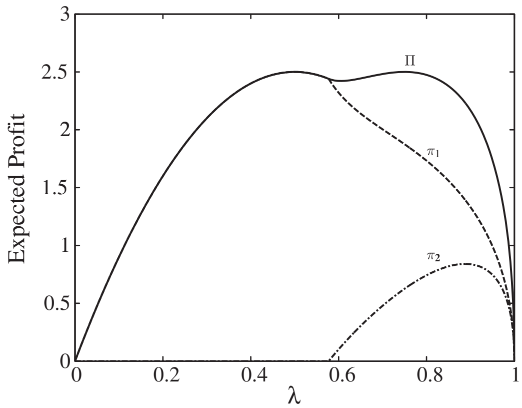

Fig. 6. The CDI's profit.

is that if possible, the CDI always maximizes the information accuracy for the high-quality service, *S*. It is not optimal for CDIs to restrict the length of transaction history data for use for the high-quality services. As CDIs collect more consumer data over time, they should improve the accuracy of its target services using all available data when the cost of data storage and processing is controllable.

Fig. 6 depicts the CDI's profit. As λ increases, it is more likely that the consumer is valuable. However, the CDI's profit's is not always increasing in λ even though the CDI's profit comes from the valuable consumer via the firms. Another important factor which influences the CDI's profit is the competition intensity between firms. When $\lambda<\frac{\sqrt{3}}{3}$, the CDI only serves firm 1 exclusively and firm 2 has no information. Firm 1's information advantage is increasing in λ when $\lambda\in\left[0,\frac{1}{2}\right]$ and decreasing in λ when $\lambda\in\left(\frac{1}{2},\frac{\sqrt{3}}{3}\right]$. Note that when $\lambda=\frac{1}{2}$, the market is most uncertain to the uninformed firm (i.e., firm 2) and the firms' competition intensity is lowest. Therefore, the CDI's profit is quasiconcave in λ, achieving the maximum level when $\lambda=\frac{1}{2}$.

When $\lambda>\frac{\sqrt{3}}{3}$, we have $Pr(h)>\frac{\sqrt{3}}{3}$. The CDI serves both firms and chooses a positive ζ_0 in differentiating the services to firms. The information disadvantage of firm 2 is mitigated as λ increases because the market becomes less uncertain and the CDI also provides the informative segmentation service to firm 2. This hurts firm 1's expected revenue. But the CDI's profit may be increasing in λ because firm 2 is able to make a positive profit, which may bring up the total profit, in the competition. As λ approaches one, firm 1's information advantage diminishes because the market has little uncertainty. Consequently, the CDI's profit approaches zero.

5. Concluding remarks

This paper studies the case where two firms compete in acquiring prospective customers using promotional incentives, and a CDI can provide services of customer segmentation to help firms better identify the value of customers. The results show that even when firms compete for common-value customers, they may not always compete more aggressively when they have more similar customer information. This feature of market competition provides the CDI an opportunity to serve competing firms. When the data service of the CDI focuses on revealing the consumers' attitudes towards the products (i.e., vertical preferences) not the brands (i.e., the horizontal preferences), e.g., in the markets for new products/services, the CDI can still soften the competition by providing firms with more similar information about the prospective customers.

This study also shows how the CDI can use service differentiation to endogenously create heterogeneity between firms and influence firm competition. Prior analytical study has examined the use of service exclusivity to differentiate firms [4]. This study considers both service exclusivity and service differentiation. By providing services with different informational accuracy, the CDI can fine-tune the degree of firm differentiation when it serves multiple firms. The analysis in this study suggests that the CDI may adopt service exclusivity or service differentiation under different market conditions. Specifically, in unpromising markets where the majority of customers are non-valuable, the analysis suggests that it is better for the CDI to use service exclusivity. In promising markets where the majority of customers are valuable, it is better for the CDI to serve both firms with differentiated information services. The use of both service exclusivity and service differentiation provides the CDI more flexibility to influence the firms' competition.

The study also provides many other opportunities for future research. First, this study presents the insight on how the CDI uses its service to endogenously differentiate between competing firms. Future research may incorporate the exogenous horizontal differentiation between firms and consider the CDI's service strategies for *ex ante* heterogeneous firms. Such analysis with various types of firm differentiation may generate additional insights on the CDI's service design. Second, future research may empirically test the relationship between the market conditions and the CDI's business strategies, as predicted by this study. Third, future research may consider target marketing instruments other than targeted promotional incentives, such as targeted advertising and targeted lowest-price guarantee. Studies on the mix of these marketing strategies could generate important insights on how CDIs influence firm competition.

Appendix A. Proof

A.1. Proof of remark

$$E\left[V|l\right]\leq E\left[V|\tilde{l}\right]\leq E\left[V\right]\leq E\left[V|\tilde{h}\right]\leq E\left[V|h\right]$$

Proof. The customer's expected values given $S=h$ and $S=l$ are respectively

$$E[V|h]=RP(H|h)\ \text{and}\ E[V|l]=RP(H|l).$$

Since $\psi>\phi$, we have that $P(H|h)>P(H|l)$ and $E[V|h]\geq E[V|l]$. Also, since $E[V]=Pr(h)E[V|h]+Pr(l)E[V|l]$, we have

$$E[V|l]\leq E[V]\leq E[V|h]. \tag{A.1}$$

Similarly, for firm 2, we have the following expected values

$$E\left[V|\tilde{h}\right] = P\left(h|\tilde{h}\right)E\left[V|h\right] + P\left(l|\tilde{h}\right)E\left[V|l\right],$$
$$E\left[V|\tilde{l}\right] = P\left(h|\tilde{l}\right)E\left[V|h\right] + P\left(l|\tilde{l}\right)E\left[V|l\right].$$

where $P\left(h|\tilde{h}\right)$, $P\left(h|\tilde{l}\right)$, $P\left(l|\tilde{h}\right)$, and $P\left(l|\tilde{l}\right)$ are given by

$$P\left[h|\tilde{h}\right] = \frac{Pr\left(\tilde{h}|h\right)Pr(h)}{Pr\left(\tilde{h}|h\right)Pr(h) + Pr\left(\tilde{h}|l\right)Pr(l)} = \frac{(1+\zeta)(\psi\lambda + \phi(1-\lambda))}{(1+\zeta)(\psi\lambda + \phi(1-\lambda)) + (1-\zeta)(1-\psi\lambda-\phi(1-\lambda))},$$

$$P\left[l|\tilde{h}\right] = 1-P\left[h|\tilde{h}\right],$$

$$P\left[h|\tilde{l}\right] = \frac{Pr\left(\tilde{l}|h\right)Pr(h)}{Pr\left(\tilde{l}|h\right)Pr(h) + Pr\left(\tilde{l}|l\right)Pr(l)} = \frac{(1-\zeta)(\psi\lambda + \phi(1-\lambda))}{(1-\zeta)(\psi\lambda + \phi(1-\lambda)) + (1+\zeta)(1-\psi\lambda-\phi(1-\lambda))},$$

$$P\left[l|\tilde{l}\right] = 1-P\left[h|\tilde{l}\right].$$

Since $E[V|h] \geq E[V|l]$, we have

$$E\left[V|l\right] \leq E\left[V|\tilde{h}\right] \leq E\left[V|h\right] \text{ and } E\left[V|l\right] \leq E\left[V|\tilde{l}\right] \leq E\left[V|h\right]. \tag{A.2}$$

Comparing $P\left[h|\tilde{h}\right]$ and $P\left[h|\tilde{l}\right]$, we have $P\left[h|\tilde{h}\right] \geq P\left[h|\tilde{l}\right]$ and therefore $E\left[V|\tilde{h}\right] \geq E\left[V|\tilde{l}\right]$. Since $E[V] = Pr\left(\tilde{h}\right)E\left[V|\tilde{h}\right] + Pr\left(\tilde{l}\right)E\left[V|\tilde{l}\right]$, we have

$$E\left[V|\tilde{l}\right] \leq E\left[V\right] \leq E\left[V|\tilde{h}\right]. \tag{A.3}$$

Combining Eqs. (A.1), (A.2) and (A.3), we have

$$E\left[V|l\right] \leq E\left[V|\tilde{l}\right] \leq E\left[V\right] \leq E\left[V|\tilde{h}\right] \leq E\left[V|h\right].$$

A.2. Proof of no pure-strategy equilibrium

Proof. Suppose that firm 1 and firm 2 offer deterministic promotional incentives. Let m_h and m_l denote firm 1's promotional incentive to its high-segment customer and low-segment customer, respectively. Let $m_{\tilde{h}}$ and $m_{\tilde{l}}$ denote firm 2's promotional incentive to its high-segment customer and low-segment customer, respectively. According to Remark 1, the expected value of the prospective customer is at most $E[V|h]$. Therefore, we have $m_S \leq E[V|h]$ ($S = \{h,l\}$) and $m_{\tilde{S}} \leq E[V|h]$ ($\tilde{S} = \left\{\tilde{h}, \tilde{l}\right\}$). Also, the expected value of the prospective customer is at least $E[V|l]$. Since firms compete for the customer, we have $m_S \geq E[V|l]$ and $m_{\tilde{S}} \geq E[V|l]$.

Next we consider all possible cases that firms offer deterministic promotional incentives and show that none of them can be an equilibrium.

(1) Suppose that firm 2's deterministic promotional incentives satisfy that $E[V|l] \leq m_{\tilde{l}} < m_{\tilde{h}} \leq E[V|h]$. Firm 1's best response promotional incentives are $m_h = m_{\tilde{h}} + \varepsilon$ or $m_{\tilde{l}} + \varepsilon$, and $m_l < m_{\tilde{l}}$. When firm 1 responds with $m_h = m_{\tilde{h}} + \varepsilon$, firm 2 wins the customer only when the expected value of the customer is $E[V|l]$, and thus firm 2's expected profit is negative. When firm 1 responds with $m_h = m_{\tilde{l}} + \varepsilon$, firm 2 can always increase its expected profit by decreasing $m_{\tilde{h}}$ and just beating m_h. Therefore, such deterministic $\left(m_{\tilde{h}}, m_{\tilde{l}}\right)$ cannot be in any equilibrium;

(2) Suppose that firm 2's deterministic promotional incentives satisfy that $E[V|l] \leq m_{\tilde{h}} < m_{\tilde{l}} \leq E[V|h]$. Firm 1's best-response promotional incentives are $m_h = m_{\tilde{h}} + \varepsilon$ or $m_{\tilde{l}} + \varepsilon$, and $m_l < m_{\tilde{h}}$. When firm 1 responds with $m_h = m_{\tilde{l}} + \varepsilon$, firm 2 wins the customer only when the expected value of the customer is $E[V|l]$, and thus its expected profit is negative. If firm 1 responds with $m_h = m_{\tilde{h}} + \varepsilon$, firm 2 can always be better off by decreasing $m_{\tilde{l}}$ and just beating m_h. Therefore, such deterministic $\left(m_{\tilde{h}}, m_{\tilde{l}}\right)$ cannot be in any equilibrium;

(3) Suppose that firm 2's deterministic promotional incentives satisfy that $E[V|l] < m_{\tilde{h}} = m_{\tilde{l}} = \tilde{m} < E[V|h]$. Firm 1's best-response promotional incentives are $m_h = \tilde{m} + \varepsilon$, and $m_l < \tilde{m}$. Firm 2 wins the customer only when the expected value of the customer is $E[V|l]$, and thus its expected profit is negative. Therefore, such deterministic $\left(m_{\tilde{h}}, m_{\tilde{l}}\right)$ cannot be in any equilibrium;

(4) Suppose that firm 2's deterministic promotional incentives satisfy that $m_{\tilde{h}} = m_{\tilde{l}} = E[V|h]$. Firm 1's best-response promotional incentives are $m_h \leq E[V|h]$, and $m_l < E[V|h]$. In this case, firm 2's expected profit is always negative. Therefore, such deterministic $\left(m_{\tilde{h}}, m_{\tilde{l}}\right)$ cannot be in any equilibrium;

(5) Suppose that firm 2's deterministic promotional incentives satisfy that $m_{\tilde{h}} = m_{\tilde{l}} = E[V|l]$. Firm 1's best-response promotional incentives are $m_h = E[V|l] + \varepsilon$, and $m_l \leq E[V|l]$. firm 2 can always be better off by raising $m_{\tilde{h}}$ and just beating m_h. Therefore, such deterministic $\left(m_{\tilde{h}}, m_{\tilde{l}}\right)$ cannot be in any equilibrium;

(6) Suppose that firm 1's deterministic promotional incentives satisfy that $m_h = m_l = E[V|l]$. Firm 2's best-response promotional incentives are $m_{\tilde{h}} = m_{\tilde{l}} = E[V|l] + \varepsilon$. Firm 1 can always be better off by raising m_h and just beating firm 2's best-response. Therefore, such deterministic (m_h, m_l) cannot be in any equilibrium;

(7) Suppose that firm 1's deterministic promotional incentives satisfy that $E[V|l]<m_h<E\left[V\middle|\tilde{h}\right]$, and $m_l=E[V|l]$. Firm 2's best-response promotional incentives are $m_{\tilde{h}}=m_h+\varepsilon$ or $E[V|l]$, and $p_{\tilde{l}}=m_h+\varepsilon$ or $E[V|l]$. When firm 2 responds with $m_{\tilde{h}}=m_h+\varepsilon$ or $m_{\tilde{l}}=p_h+\varepsilon$, firm 1 can always be better off by raising m_h and overbidding $m_{\tilde{h}}$ and $m_{\tilde{l}}$. When firm 2 responds with $m_{\tilde{h}}=m_{\tilde{l}}=E[V|l]$, firm 1 can always be better off by lowering m_h and just overbidding $m_{\tilde{h}}$ and $m_{\tilde{l}}$. Therefore, such deterministic (m_h,m_l) cannot be in any equilibrium;

(8) Suppose that firm 1's deterministic promotional incentives satisfy that $E\left[V\middle|\tilde{h}\right]\le m_h\le E[V|h]$, and $m_l=E[V|l]$. Firm 2's best-response promotional incentives are $m_{\tilde{h}}=m_{\tilde{l}}=E[V|l]$. Firm 1 can always be better off by decreasing m_h and just beating $m_{\tilde{h}}$ and $m_{\tilde{l}}$. Therefore, such deterministic (m_h,m_l) cannot be in any equilibrium.

Considering (1)-(8), we conclude that there is no pure-strategy equilibrium.

A.3. Proof of Proposition 1

Proof. Let $\pi_1(m_1|h)$ and $\pi_1(m_1|l)$ denote firm 1's expected profits of offering a promotional incentive m_1 for its high-segment customer $(S=h)$ and its low-segment customer $(S=l)$ respectively. Let $\pi_2\left(m_2\middle|\tilde{h}\right)$ and $\pi_2\left(m_2\middle|\tilde{l}\right)$ denote firm 2's expected profit of offering a promotional incentive m_2 for its high-segment customer $(\tilde{S}=\tilde{h})$ and its low-segment customer $(\tilde{S}=\tilde{l})$ respectively.

For the mixed-strategy equilibrium, we first consider the support ranges of firms' randomized promotional incentives. Let $\overline{m}_{1S}$ and $\underline{m}_{1S}$ denote firm 1's upper bound and lower bound of promotional incentives respectively for its customer in the two segments, $S\in\{h,l\}$. Let $\overline{m}_{2\tilde{S}}$ and $\underline{m}_{2\tilde{S}}$ denote firm 2's upper bound and lower bound of promotional incentives respectively for its customer in the two segments, $\tilde{S}\in\left\{\tilde{h},\tilde{l}\right\}$. Regarding the values of these bounds, we can immediately conclude three *boundary conditions*:

Boundary Condition 1: $\underline{m}_{1S}\ge E[V|l]$, $\underline{m}_{2\tilde{S}}\ge E[V|l]$. Since the lowest expected value is $E[V|l]$ and firms compete with each other to win the consumer, no one will offer lower than $E[V|l]$.
Boundary Condition 2: $\overline{m}_{1S}\le E[V|S]$, $\overline{m}_{2\tilde{S}}\le E\left[V\middle|\tilde{S}\right]$. That is, a firm will not offer a promotional incentive higher than its expected value of the customer.
Boundary Condition 3: $max\{\overline{m}_{1h},\overline{m}_{1l}\}=max\{\overline{m}_{2\tilde{h}},\overline{m}_{2\tilde{l}}\}$.

Lemma A1. *Firm 1 always offers a deterministic promotional incentive E[V|l] for its low-segment customer (S=l). In other word,* $\underline{m}_{1l}=\overline{m}_{1l}=E[V|l]$.

Proof. For its low-segment customer $(S=l)$, firm 1's expected value of the customer is $E[V|l]$. Firm 1 will not offer any promotional incentive above $E[V|l]$. That is, $\overline{m}_{1l}\le E[V|l]$. Boundary Condition 1 indicates that $\underline{m}_{1l}\ge E[V|l]$. Therefore, we can conclude that $\underline{m}_{1l}=\overline{m}_{1l}=E[V|l]$ and firm 1 offers a deterministic promotional incentive to its low-segment customer $(S=l)$. That is, $G_1(m|l)=\{0, m<E[V|l] 1, m\ge E[V|l]$. In this case, firm 1 makes a zero expected profit from the low-segment, i.e., $\pi_1(m_1|l)=0$.

Using the result of Lemma A1, we can simplify Boundary Condition 3 to $\overline{m}_{1h}=max\{\overline{m}_{1h},\overline{m}_{1l}\}=max\{\overline{m}_{2\tilde{h}},\overline{m}_{2\tilde{l}}\}$ and the firms' profit functions as

$$\pi_1(m_1|h)=Pr\left(\tilde{h}|h\right)G_2\left(m_1\middle|\tilde{h}\right)\left(E[V|h]-m_1\right)+Pr\left(\tilde{l}|h\right)G_2\left(m_1\middle|\tilde{l}\right)(E[V|h]-m_1), \tag{A.4}$$

$$\pi_2\left(m_2\middle|\tilde{h}\right)=P\left(h\middle|\tilde{h}\right)G_1(m_2|h)(E[V|h]-m_2)+P\left(l\middle|\tilde{h}\right)(E[V|l]-m_2), \tag{A.5}$$

$$\pi_2\left(m_2\middle|\tilde{l}\right)=P\left(h\middle|\tilde{l}\right)G_1(m_2|h)(E[V|h]-m_2)+P\left(l\middle|\tilde{l}\right)(E[V|l]-m_2). \tag{A.6}$$

Lemma A2. *Firm 2's upper bound of the promotional incentive for its low-segment customer* $(\tilde{S}=\tilde{l})$ *is also its lower bound of the promotional incentive for its high-segment customer* $(\tilde{S}=\tilde{h})$. *We define this level as* $\hat{m}$, *i.e.*, $\hat{m}\triangleq\underline{m}_{2\tilde{h}}=\overline{m}_{2\tilde{l}}$.

Proof. We first prove that we cannot have $\underline{m}_{2\tilde{h}}<\overline{m}_{2\tilde{l}}$ and then prove that we cannot have $\underline{m}_{2\tilde{h}}>\overline{m}_{2\tilde{l}}$.

Suppose $\underline{m}_{2\tilde{h}}<\overline{m}_{2\tilde{l}}$. The supports of $G_2\left(m|\tilde{h}\right)$ and $G_2\left(m|\tilde{l}\right)$ overlap on $[\underline{m}_{2\tilde{h}},\overline{m}_{2\tilde{l}}]$. In the mixed-strategy equilibrium, from a high-segment customer $(\tilde{S}=\tilde{h})$, firm 2 always makes a constant expected profit $\pi_2\left(m_2|\tilde{h}\right)$ when offering any promotional incentive $m_2\in[\underline{m}_{2\tilde{h}},\overline{m}_{2\tilde{l}}]$. Similarly, from a low-segment customer $(\tilde{S}=\tilde{l})$, firm 2 always makes a constant expected profit $\pi_2\left(m_2|\tilde{l}\right)$ when offering any promotional incentive $m_2\in[\underline{m}_{2\tilde{h}},\overline{m}_{2\tilde{l}}]$. Therefore, $\pi_2\left(m_2|\tilde{h}\right)-\pi_2\left(m_2|\tilde{l}\right)$ should be constant for any $m_2\in[\underline{m}_{2\tilde{h}},\overline{m}_{2\tilde{l}}]$.

$$\begin{aligned}\pi_2(m_2|\tilde{h})-\pi_2(m_2|\tilde{l})&=\left(P\left(h|\tilde{h}\right)-P\left(h|\tilde{l}\right)\right)G_1(m_2|h)(E[V|h]-m_2)\\&\quad+\left(P\left(l|\tilde{h}\right)-P\left(l|\tilde{l}\right)\right)(E[V|l]-m_2)\\&=\left(P\left(h|\tilde{h}\right)-P\left(h|\tilde{l}\right)\right)\begin{pmatrix}G_1(m_2|h)E[V|h](7)\\+(1-G_1(m_2|h))m_2-E[V|l]\end{pmatrix}.\end{aligned} \tag{A.7}$$

$\pi_2\left(m_2|\tilde{h}\right)-\pi_2\left(m_2|\tilde{l}\right)$ being constant requires that $G_1(m_2|h)E[V|h]+(1-G_1(m_2|h))m_2-E[V|l]$ is also constant. Suppose that $G_1(m_2|h)E[V|h]+(1-G_1(m_2|h))m_2-E[V|l]=C$ for any $m_2\in[\underline{m}_{2\tilde{h}},\overline{m}_{2\tilde{l}}]$, we have $G_1(m_2|h)=\frac{C+E[V|l]-m_2}{E[V|h]-m_2}$ and $\frac{dG_1(m_2|h)}{dm_2}=\frac{C+E[V|l]-E[V|h]}{(E[V|h]-m_2)^2}$. Since $m_2<E[V|h]$, we have

$G_1(m_2|h)E[V|h]+(1-G_1(m_2|h))m_2<E[V|h]$ and hence $C<E[V|h]-E[V|l]$. Therefore, $\frac{dG_1(m_2|h)}{dm_2}=\frac{C+E[V|l]-E[V|h]}{(E[V|h]-m_2)^2}<0$, which indicates that the cumulative probability function $G_1(m_2|h)$ is decreasing. However, this cannot be true since a cumulative probability function is nondecreasing. Therefore, we cannot have $\underline{m}_{2\tilde{h}}<\overline{m}_{2\tilde{l}}$.

We next prove that we cannot have $\underline{m}_{2\tilde{h}}>\overline{m}_{2\tilde{l}}$. Suppose $\underline{m}_{2\tilde{h}}>\overline{m}_{2\tilde{l}}$. When $\underline{m}_{2\tilde{h}}>\overline{m}_{2\tilde{l}}$, there is a gap between the supports for $G_2\left(m|\tilde{h}\right)$ and $G_2\left(m|\tilde{l}\right)$. As a result, firm 1 will never offer a promotional incentive $m_1\in\left(\overline{m}_{2\tilde{l}},\underline{m}_{2\tilde{h}}\right)$. Firm 1 will not offer $m_1=\underline{m}_{2\tilde{h}}$ either. This is because $\overline{m}_{2\tilde{l}}$ and $\underline{m}_{2\tilde{h}}$ lead to the same probability of winning but firm 1 pays more at $\underline{m}_{2\tilde{h}}$. As a result, firm 2 will also never offer $m_2=\underline{m}_{2\tilde{h}}$, which contradicts the fact that $\underline{m}_{2\tilde{h}}$ is firm 2's lower bound of randomization. Therefore, we cannot have $\underline{m}_{2\tilde{h}}>\overline{m}_{2\tilde{l}}$.

Overall, we exclude both $\underline{m}_{2\tilde{h}}<\overline{m}_{2\tilde{l}}$ and $\underline{m}_{2\tilde{h}}>\overline{m}_{2\tilde{l}}$. The only possibility is $\underline{m}_{2\tilde{h}}=\overline{m}_{2\tilde{l}}$.

Lemma A3. *$\hat{m}$ is not lower than the lower bound of promotional incentive distribution of firm 1 for its high-segment customer ($S=h$), i.e., $\hat{m}\geq\underline{m}_{1h}$.*

Proof. Suppose $\hat{m}<\underline{m}_{1h}$. If firm 2 offers any promotional incentive $m_2\in(\hat{m},\underline{m}_{1h})$, it only wins the customer when the customer is classified as a low-segment customer for firm 1 ($S=l$) and hence earns a negative profit. Therefore, $\hat{m}\geq\underline{m}_{1h}$.

Lemma A4. *Firm 1's upper bound of promotional incentive for its high-segment customer ($S=h$) is equal to firm 2's upper bound of promotional incentive for firm 2's high-segment ($\tilde{S}=\tilde{h}$). We define this level as $\overline{m}_1$. There is no mass point at $\overline{m}_1$ for both firms' promotional incentive distributions. That is, $\overline{m}_1\triangleq\overline{m}_{1h}=\overline{m}_{2\tilde{h}}$ and $G_1(\overline{m}_1|h)=G_2\left(\overline{m}_1|\tilde{h}\right)=1$.*

Proof. It is straightforward that $\overline{m}_{2\tilde{h}}=\overline{m}_{1h}$. We define $\overline{m}_1\triangleq\overline{m}_{1h}=\overline{m}_{2\tilde{h}}$. From Boundary Condition 2, we have $\overline{m}_1\leq E\left[V|\tilde{h}\right]$.

We next prove that firms put no mass point at $\overline{m}_1$. Suppose that firm 2 puts a mass point at $\overline{m}_1$. Firm 1 has an incentive to increase m_1 to $\overline{m}_1+\varepsilon$ to beat firm 2. Therefore, $\overline{m}_1$ is not the upper bound of firms' promotional incentives.

Suppose firm 2 does not have a mass point at $\overline{m}_1$ but firm 1 does. We have $G_1(\overline{m}_1|h)<1$. If $\overline{m}_1<E\left[V|\tilde{h}\right]$, firm 2 has an incentive to increase m_2 to $\overline{m}_1+\varepsilon$ to beat firm 1. Therefore, $\overline{m}_1$ is not the upper bound of firms' promotional incentives. If $\overline{m}_1=E\left[V|\tilde{h}\right]$, firm 2's expected profit when offering $\overline{m}_1$ is

$$\begin{aligned}\pi_2\left(\overline{m}_1|\tilde{h}\right)&=P\left(h|\tilde{h}\right)G_1(\overline{m}_1|h)(E[V|h]-\overline{m}_1)+P\left(l|\tilde{h}\right)(E[V|l]-\overline{m}_1),\\&<P\left(h|\tilde{h}\right)(E[V|h]-\overline{m}_1)+P\left(l|\tilde{h}\right)(E[V|l]-\overline{m}_1),\\&=E\left[V|\tilde{h}\right]-\overline{m}_1\\&=0.\end{aligned}$$

In other words, firm 2's expected profit is negative. Therefore, we cannot have firm 1 putting a mass point at $\overline{m}_1$ either.

Overall, we have $G_1(\overline{m}_1|h)=G_2\left(\overline{m}_1|\tilde{h}\right)=1$.

So far, we can conclude that:

1. The support for $G_2\left(m|\tilde{l}\right)$ is $[\underline{m}_{2\tilde{l}},\hat{m}]$;
2. The support for $G_2\left(m|\tilde{h}\right)$ is $[\hat{m},\overline{m}_1]$;
3. The support for $G_1(m|h)$ is $[\underline{m}_{1h},\overline{m}_1]$.

We next consider firm 2's expected profit in competition. We first show that $\pi_2\left(m|\tilde{l}\right)=0$, i.e., firm 2 cannot make a positive expected profit when $\tilde{S}=\tilde{l}$. We next consider firm 2's expected profit when $\tilde{S}=\tilde{h}$. Prior literature suggests that when competing with a better-informed competitor, the less-informed competitor earns a zero expected profit in the mixed-strategy equilibrium (e.g. Engelbrecht-Wiggans et al. 1983). We use this key finding to form two conjectures on the expected profits of firm 2 (the less informed firm) when $\tilde{S}=\tilde{h}$.

Lemma A5.

$$\pi_2\left(m|\tilde{l}\right)=0.$$

Proof. Suppose $\pi_2\left(m|\tilde{l}\right)>0$. From Eq. (A.6), we have

$$\pi_2\left(\underline{m}_{2\tilde{l}}|\tilde{l}\right)=P\left(h|\tilde{l}\right)G_1\left(\underline{m}_{2\tilde{l}}|h\right)\left(E[V|h]-\underline{m}_{2\tilde{l}}\right)+P\left(l|\tilde{l}\right)\left(E[V|l]-\underline{m}_{2\tilde{l}}\right)>0.$$

Considering Boundary Conditions 1 and 2, we have $P\left(l|\tilde{l}\right)\left(E[V|l]-\underline{m}_{2\tilde{l}}\right)\leq0$. Therefore, to satisfy $\pi_2\left(\underline{m}_{2\tilde{l}}|\tilde{l}\right)>0$, we must have $G_1\left(\underline{m}_{2\tilde{l}}|h\right)>0$. $G_1\left(\underline{m}_{2\tilde{l}}|h\right)$ is positive only when either of the following cases is true: (1) $\underline{m}_{1h}<\underline{m}_{2\tilde{l}}$; (2) $\underline{m}_{1h}=\underline{m}_{2\tilde{l}}$ and firm 1 has a mass point at $\underline{m}_{2\tilde{l}}$.

We next show that the first case $\underline{m}_{1h}<\underline{m}_{2\tilde{l}}$ cannot be true. If $\underline{m}_{1h}<\underline{m}_{2\tilde{l}}$ is true, firm 1 makes a zero expected profit when offering $m_1=\underline{m}_{1h}$ and $S=h$. However, Boundary Condition 2 suggests that firm 2 always offers a promotional incentive $m_2\leq E\left[V|\tilde{h}\right]$, which is lower than $E[V|h]$. When $S=h$, firm 1 can make a positive profit at least by offering $m_1=E\left[V|\tilde{h}\right]+\varepsilon$. Therefore, firm 1 should make a positive profit at $\underline{m}_{1h}$ when $S=h$. Therefore, the case, $\underline{m}_{1h}<\underline{m}_{2\tilde{l}}$, cannot be true.

We next show that the second case, i.e., $\underline{m}_{1h} = \underline{m}_{2\tilde{l}}$ and firm 1 has a mass point at $\underline{m}_{2\tilde{l}}$ cannot be true. Because firms cannot both have a mass point at a boundary simultaneously in equilibrium, if firm 1 has a mass point at $\underline{m}_{1h} = \underline{m}_{2\tilde{l}}$, we must have $G_2\left(\underline{m}_{1h}|\tilde{h}\right) = 0$ and $G_2\left(\underline{m}_{1h}|\tilde{l}\right) = 0$. Considering Eq. (A.4), we have

$$\pi_1\left(\underline{m}_{1h}|h\right) = Pr\left(\tilde{h}|h\right)G_2\left(\underline{m}_{1h}|\tilde{h}\right)\left(E[V|h] - \underline{m}_{1h}\right) + Pr\left(\tilde{l}|h\right)G_2\left(\underline{m}_{1h}|\tilde{l}\right)\left(E[V|h] - \underline{m}_{1h}\right).$$

If $G_2\left(\underline{m}_{1h}|\tilde{h}\right) = 0$ and $G_2\left(\underline{m}_{1h}|\tilde{l}\right) = 0$, we must have that $\pi_1\left(\underline{m}_{1h}|h\right)$is always zero and therefore firm 1's expected profit when $S=h$ is always zero. However, this cannot be true as we proved above. Therefore, the second case cannot be true.

In conclusion, firm 2 must make a zero expected profit when $\tilde{S} = \tilde{l}$, i.e., $\pi_2\left(m|\tilde{l}\right) = 0$.

Next, we develop the following conjectures:

Conjecture 1: For $m_2 \in \left[\underline{m}_{2\tilde{l}}, \hat{m}\right], \pi_2\left(m_2|\tilde{l}\right) = 0$ and for $m_2 \in [\hat{m}, \overline{m}_1],\ \pi_2\left(m_2|\tilde{h}\right) = 0$. In other words, firm 2's expected profits from its low-segment customer ($\tilde{S} = \tilde{l}$) and high-segment customer ($\tilde{S} = \tilde{h}$) are *always zero*;

Conjecture 2: For $m_2 \in \left[\underline{m}_{2\tilde{l}}, \hat{m}\right],\ \pi_2\left(m_2|\tilde{l}\right) = 0$ and for $m_2 \in [\hat{m}, \overline{m}_1],\ \pi_2\left(m_2|\tilde{h}\right) > 0$. In other words, firm 2's expected profit from its low-segment customer ($\tilde{S} = \tilde{l}$) is *always zero*, but that from its high-segment customer ($\tilde{S} = \tilde{h}$) *may be zero or positive*.

We will first use Conjecture 1 to derive the equilibrium in Proposition 1. Conjecture 2 will be used to derive the equilibrium in Proposition 2.

Based on Conjecture 1, we have $\pi_2\left(m|\tilde{h}\right) = P\left(h|\tilde{h}\right)G_1(m|h)(E[V|h] - m) + P\left(l|\tilde{h}\right)(E[V|l] - m) = 0$ (please see Eq. (5)), we have

$$G_1(m|h) = \frac{P\left(l|\tilde{h}\right)(m - E[V|l])}{P\left(h|\tilde{h}\right)(E[V|h] - m)}. \tag{A.8}$$

Lemma A6.

$$\overline{m}_1 = E\left[V\middle|\tilde{h}\right].$$

Proof. By Lemma A4 and (A.8), we have $G_1(\overline{m}_1|h) = \frac{P\left(l|\tilde{h}\right)(\overline{m}_1 - E[V|l])}{P\left(h|\tilde{h}\right)(E[V|h] - \overline{m}_1)} = 1$. Therefore, $\overline{m}_1 = P\left(h|\tilde{h}\right)E[V|h] + P\left(l|\tilde{h}\right)E[V|l] = E\left[V|\tilde{h}\right]$.

Lemma A7. $\hat{m} = \overline{m}_{2\tilde{l}} = \underline{m}_{2\tilde{h}} = \underline{m}_{1h} = E[V|l]$ and $G_1(m|h)|_{m=E[V|l]} = 0$.

Proof. Suppose $\underline{m}_{1h} > E[V|l]$. From Lemma A3, we must have $\hat{m} \geq \underline{m}_{1h} > E[V|l]$. We plug (A.8) into (A.6) and replace m with $\hat{m}$, we have

$$\begin{aligned}\pi_2\left(\hat{m}|\tilde{l}\right) &= P\left(h|\tilde{l}\right)\frac{P\left(l|\tilde{h}\right)(\hat{m} - E[V|l])}{P\left(h|\tilde{h}\right)(E[V|h] - \hat{m})}(E[V|h] - \hat{m}) + P\left(l|\tilde{l}\right)(E[V|l] - \hat{m}) \\ &= \left(\frac{P\left(h|\tilde{l}\right)P\left(l|\tilde{h}\right) - P\left(l|\tilde{l}\right)P\left(h|\tilde{h}\right)}{P\left(h|\tilde{h}\right)}\right)(\hat{m} - E[V|l]).\end{aligned}$$

Because $P\left(h|\tilde{l}\right)P\left(l|\tilde{h}\right) < P\left(l|\tilde{l}\right)P\left(h\middle|\tilde{h}\right)$, we have $\pi_2\left(\hat{m}|\tilde{l}\right) < 0$. This contradicts the fact that $\pi_2\left(\hat{m}|\tilde{l}\right) \geq 0$. To ensure that $\pi_2\left(\hat{m}|\tilde{l}\right) = 0$, we must have $\hat{m} = E[V|l]$. Then we also have $\underline{m}_{1h} = \underline{m}_{2\tilde{h}} = \overline{m}_{2\tilde{l}} = E[V|l]$. It is easy to verify that $G_1\left(\underline{m}_{1h}|h\right) = 0$. Therefore, $G_1(m|h)$ has no mass point at $\underline{m}_{1h}$. We can conclude that firm 2 offers $m_2 = E[V|l]$ to its low-segment customer ($S = \tilde{l}$). That is, $G_2\left(m|\tilde{l}\right) = \{0, m < E[V|l]\, 1,\ m \geq E[V|l]$.

We can rewrite firm 1's expected profit from its high-segment customer, given any promotional incentive, as

$$\pi_1(m_1|h) = Pr\left(\tilde{h}|h\right)G_2\left(m_1|\tilde{h}\right)(E[V|h] - m_1) + Pr\left(\tilde{l}|h\right)(E[V|h] - m_1). \tag{A.9}$$

Using Lemma A4 and A6, we can conclude that firm 1's expected profit is $\pi_1(m_1|h) = E[V|h] - E\left[V|\tilde{h}\right]$ from its high-segment customer. Therefore, we have

$$G_2\left(m|\tilde{h}\right) = \frac{E[V|h] - E\left[V|\tilde{h}\right] - Pr\left(\tilde{l}|h\right)(E[V|h] - m)}{Pr\left(\tilde{h}|h\right)(E[V|h] - m)}. \tag{A.10}$$

Combining the results in Lemma A1, Lemma A7, Eq. (9), and Eq. (11), when $Pr(h) \leq Pr(l)$ and $\zeta < 1$, the firms' equilibrium strategies can be characterized as follows:

1. For its low-segment customer ($S = l$), firm 1 offers $E[V|l]$;
2. For its low-segment customer ($\tilde{S} = \tilde{l}$), firm 2 offers $E[V|l]$;

3. For its high-segment customer ($S=h$), the CDF of firm 1's promotional incentive is

$$G_1(m|h) = \frac{P\left(l|\tilde{h}\right)(m-E[V|l])}{P\left(h|\tilde{h}\right)(E[V|h]-m)}, m\in\left[E[V|l], E\left[V|\tilde{h}\right]\right];$$

4. For its high-segment customer ($S=\tilde{h}$), the CDF of firm 2's promotional incentive is

$$G_2\left(m|\tilde{h}\right) = \frac{E[V|h]-E\left[V|\tilde{h}\right]-Pr\left(\tilde{l}|h\right)(E[V|h]-m)}{Pr\left(\tilde{h}|h\right)(E[V|h]-m)}, m\in\left[E[V|l], E\left[V|\tilde{h}\right]\right].$$

We can verify that $G_1(m|h)\Big|_{m=E\left[V|\tilde{h}\right]}\leq 1, G_1(m|h)\big|_{m=E[V|l]}\geq 0$, $G_2\left(m|\tilde{h}\right)\Big|_{m=E\left[V|\tilde{h}\right]}\leq 1$, and $G_2\left(m|\tilde{h}\right)\Big|_{m=E[V|l]}\geq 0$ hold if and only if $Pr(h)\leq Pr(l)$. In other words, $Pr(h)\leq Pr(l)$ is a sufficient and necessary condition of the existence of this equilibrium.

A.4. Proof of Proposition 2

Proof. We consider the case $Pr(h)>Pr(l)$. In the proof of Proposition 1, we used Conjecture 1 to derive the equilibrium. We also showed that the equilibrium in Proposition 1 exists if and only if $Pr(h)\leq Pr(l)$. In the following proof, we will use Conjecture 2 to derive the equilibrium when $Pr(h)>Pr(l)$.

We will show that there exists a mixed-strategy equilibrium that firm 2 randomizes over $[\hat{m},\overline{m}]$ for its high-segment customer and over $[\underline{m}_2,\hat{m}]$ for its low-segment customer. Firm 1 randomizes over $[\underline{m}_2,\overline{m}]$ for its high-segment customer. The cutoff levels $\underline{m}_2,\hat{m}$ and $\overline{m}$ are to be determined.

Lemma A5 indicates that $\pi_2\left(\underline{m}_2|\tilde{l}\right)=0$, i.e., in the mixed-strategy equilibrium, firm 2 earns zero expected profit from its low-segment customer. From $\pi_2\left(\underline{m}_2|\tilde{l}\right)=P\left(h|\tilde{l}\right)G_1(\underline{m}_2|h)(E[V|h]-\underline{m}_2)+P\left(l|\tilde{l}\right)\times(E[V|l]-\underline{m}_2)=0$, $m\in[\underline{m}_2,\hat{m}]$, we have

$$G_1(m|h) = \frac{P\left(l|\tilde{l}\right)(m-E[V|l])}{P\left(h|\tilde{l}\right)(E[V|h]-m)}, m\in[\underline{m}_2,\hat{m}]. \tag{A.11}$$

Considering Lemma A4 and the constant expected profits of firms in the mixed-strategy equilibrium, we have

$$\begin{aligned}
&\pi_2\left(m_2|\tilde{h}\right)=P\left(h|\tilde{h}\right)G_1(m_2|h)(E[V|h]-m_2)+P\left(l|\tilde{h}\right)(E[V|l]-m_2)=E\left[V|\tilde{h}\right]-\overline{m}, m\in[\hat{m},\overline{m}],\\
&\pi_1(m_1|h)=Pr\left(\tilde{h}|h\right)G_2\left(m_1|\tilde{h}\right)(E[V|h]-m_1)+Pr\left(\tilde{l}|h\right)(E[V|h]-m_1)=E[V|h]-\overline{m}, m\in[\hat{m},\overline{m}],\\
&\pi_1(m_1|h)=Pr\left(\tilde{l}|h\right)G_2\left(m_1|\tilde{l}\right)(E[V|h]-m_1)=E[V|h]-\overline{m}, m\in[\underline{m}_2,\hat{m}].
\end{aligned}$$

We therefore can derive

$$G_1(m|h) = \frac{E\left[V|\tilde{h}\right]-\overline{m}-P\left(l|\tilde{h}\right)(E[V|l]-m)}{P\left(h|\tilde{h}\right)(E[V|h]-m)}, m\in[\hat{m},\overline{m}], \tag{A.12}$$

$$G_2\left(m|\tilde{h}\right) = \frac{E[V|h]-\overline{m}-Pr\left(\tilde{l}|h\right)(E[V|h]-m)}{Pr\left(\tilde{h}|h\right)(E[V|h]-m)}, m\in[\hat{m},\overline{m}], \tag{A.13}$$

and

$$G_2\left(m|\tilde{l}\right) = \frac{E[V|h]-\overline{m}}{Pr\left(\tilde{l}|h\right)(E[V|h]-m)}, m\in[\underline{m}_2,\hat{m}]. \tag{A.14}$$

We next derive $\hat{m}$ and $\overline{m}$. In the mixed-strategy equilibrium, we should have $\pi_1(\hat{m}|h)=\pi_1(\overline{m}|h)$ and $\pi_2\left(\hat{m}|\tilde{h}\right)=\pi_2\left(\overline{m}|\tilde{h}\right)$, which can be expressed as follows,

$$Pr\left(\tilde{l}|h\right)(E[V|h]-\hat{m})=E[V|h]-\overline{m}, \tag{A.15}$$

$$P\left(h|\tilde{h}\right)G_1(\hat{m}|h)(E[V|h]-\hat{m})+P\left(l|\tilde{h}\right)(E[V|l]-\hat{m})=E\left[V|\tilde{h}\right]-\overline{m}, \tag{A.16}$$

where $G_1(\hat{m}|h)=\frac{P\left(l|\tilde{l}\right)(\hat{m}-E[V|l])}{P\left(h|\tilde{l}\right)(E[V|h]-\hat{m})}$ from Eq. (A.12). Jointly solving Eqs. (A.15) and (A.16), we have

$$\hat{m}=\frac{P\left(h|\tilde{h}\right)P\left(l|\tilde{l}\right)E[V|l]+P\left(h|\tilde{l}\right)\left(P\left(h|\tilde{h}\right)-Pr\left(\tilde{h}|h\right)\right)E[V|h]}{\left(P\left(h|\tilde{h}\right)P\left(l|\tilde{l}\right)-P\left(h|\tilde{l}\right)P\left(l|\tilde{h}\right)+P\left(h|\tilde{l}\right)Pr\left(\tilde{l}|h\right)\right)}, \tag{A.17}$$

$$\overline{m}=\frac{\left\{\begin{array}{c}\left(Pr\left(\tilde{h}|h\right)P\left(h|\tilde{h}\right)P\left(l|\tilde{l}\right)-Pr\left(\tilde{h}|h\right)P\left(h|\tilde{l}\right)P\left(l|\tilde{h}\right)+Pr\left(\tilde{l}|h\right)P\left(h|\tilde{l}\right)P\left(h|\tilde{h}\right)\right)E[V|h]\\ +Pr\left(\tilde{l}|h\right)P\left(h|\tilde{h}\right)P\left(l|\tilde{l}\right)E[V|l]\end{array}\right\}}{P\left(h|\tilde{h}\right)P\left(l|\tilde{l}\right)-P\left(h|\tilde{l}\right)P\left(l|\tilde{h}\right)+P\left(h|\tilde{l}\right)Pr\left(\tilde{l}|h\right)}. \tag{A.18}$$

We next verify that $E[V|l]$ is the lower bound of $G_1(m|h)$ and $G_2\left(m|\tilde{l}\right)$, and only $G_2\left(m|\tilde{l}\right)$ has a mass point at $E[V|l]$. From Eqs. (A.11) and (A.13), we have

(1) When $m_1=E[V|l]$, $G_1(m_1|h)=0$ and when $m_2=E[V|l]$, $G_2\left(m_2|\tilde{l}\right)>0$;

(2) If $\underline{m}_2>E[V|l]$, we have $G_1(\underline{m}_2|h)>0$ and $G_2\left(\underline{m}_2|\tilde{l}\right)>0$. Because firms cannot both have a mass point at the lower bound, this cannot be true.

Therefore, in equilibrium, $\underline{m}_2=E[V|l]$, $G_1(\underline{m}_2|h)=0$ and $G_2\left(\underline{m}_2|\tilde{l}\right)>0$. Only firm 2 has a mass point at the lower bound.

Combining the results in Lemma A1, Eqs. (A.11), (A.12), (A.13) and (A.14), when $Pr(h)>Pr(l)$ and $\zeta<1$, the firms' equilibrium strategies can be characterized as follows:

1. For its low-segment customer ($S=l$), firm 1 offers $E[V|l]$;
2. For its low-segment customer ($\tilde{S}=\tilde{l}$), firm 2's promotional incentive follows the CDF

$$G_2\left(m|\tilde{l}\right)=\frac{E[V|h]-\overline{m}}{Pr\left(\tilde{l}|h\right)(E[V|h]-m)},\ m\in[E[V|l],\hat{m}];$$

3. For its high-segment customer ($S=h$), firm 1's promotional incentive follows the CDF

$$G_1(m|h)=\begin{cases}\dfrac{P\left(l|\tilde{l}\right)(m-E[V|l])}{P\left(h|\tilde{l}\right)(E[V|h]-m)},\ m\in[E[V|l],\hat{m}];\\[2ex] \dfrac{E\left[V|\tilde{h}\right]-\overline{m}-P\left(l|\tilde{h}\right)(E[V|l]-m)}{P\left(h|\tilde{h}\right)(E[V|h]-m)},\ m\in[\hat{m},\overline{m}];\end{cases}$$

4. For its high-segment customer ($S=\tilde{h}$), firm 2's promotional incentive follows the CDF

$$G_2\left(m|\tilde{h}\right)=\frac{E[V|h]-\overline{m}-Pr\left(\tilde{l}|h\right)(E[V|h]-m)}{Pr\left(\tilde{h}|h\right)(E[V|h]-m)},\ m\in[\hat{m},\overline{m}].$$

We can verify that $G_1(m|h)|_{m=\overline{m}}\le 1$, $G_1(m|h)|_{m=E[V|l]}\ge 0$, $G_2\left(m|\tilde{l}\right)|_{m=\hat{m}}\le 1$, $G_2\left(m|\tilde{l}\right)|_{m=E[V|l]}\ge 0$, $G_2\left(m|\tilde{h}\right)|_{m=\overline{m}}\le 1$, $G_2\left(m|\tilde{h}\right)|_{m=\hat{m}}\ge 0$, $\overline{m}\le E\left[V|\tilde{h}\right]$ and $\hat{m}>E[V|l]$ hold if and only if $Pr(h)>Pr(l)$. In other words, $Pr(h)>Pr(l)$ is a sufficient and necessary condition of the existence of the second equilibrium.

We can conclude that the equilibrium in Proposition 1 exists if and only if $Pr(h)\le Pr(l)$, and the equilibrium in Proposition 2 exists if and only if $Pr(h)>Pr(l)$. Therefore, given a pair of $Pr(h)$ and $Pr(l)$, there is only one equilibrium. In other words, these two equilibria we derived are unique.

To further confirm that the equilibria in Proposition 1 and Proposition 2 are unique equilibria, we consider a potential alternative equilibrium when $\zeta=0$. When $\zeta=0$, firm 2 has no useful information. Firm 2 can completely ignores its information in competition, i.e., randomizes promotional incentive m_2 independent on $\tilde{S}$. Using the similar analysis, we can characterize the equilibrium of firm competition as follows:

1. For its low-segment customer ($S=l$), firm 1 offers $E[V|l]$;
2. For its high-segment customer ($S=h$), firm 1's promotional incentive follows the CDF

$$G_{10}(m|h)=\frac{Pr(l)(m-E[V|l])}{Pr(h)(E[V|h]-m)},\ m\in[E[V|l],E[V]];$$

3. Firm 2's promotional incentive follows the CDF

$$G_{20}(m)=\frac{E[V|h]-E[V]}{E[V|h]-m},\ m\in[E[V|l],E[V]].$$

We can show that $G_{10}(m|h)$ is equivalent to $G_1(m|h)|_{\zeta=0}$ and $G_{20}(m|h)$ is equivalent to $\frac{1}{2}G_2\left(m|\tilde{h}\right)|\zeta=0+\frac{1}{2}G_2\left(m|\tilde{l}\right)|\zeta=0$. Therefore, we can confirm that given a pair of $Pr(h)$ and $Pr(l)$, only a unique equilibrium exists.

A.5. Proof of Proposition 3

Proof. We first consider the case when $Pr(h)\le Pr(l)$ (or equivalently, $Pr(h)\le\frac{1}{2}$). For notational simplicity, we let $\gamma=Pr(h)$. For its low-segment customer $(S=l)$, firm 1 offers a deterministic promotional incentive and its expected profit from the low-segment customer is zero. For its high-segment customer $(S=h)$, firm 1 randomizes its promotional incentive following Eq. (A.8) and its expected profit is $E[V|h]-E\left[V|\tilde{h}\right]$. Overall, when $0<\gamma\le\frac{1}{2}$ and $0\le\zeta<1$, firm 1's expected profit is

$$\pi_1=Pr(h)\left(E[V|h]-E\left[V|\tilde{h}\right]\right)=\frac{\gamma(1-\gamma)(1-\zeta)}{1-\zeta+2\zeta\gamma}(E[V|h]-E[V|l])>0. \tag{A.19}$$

$\pi_1=0$ when $\gamma=0$, i.e., the customer will never be classified in the high segment for firm 1. This case is trivial. We ignore the discussion on this case in the study. We generally say firm 1 makes a positive profit when $Pr(h)\le Pr(l)$. Firm 2's expected profit from a customer is always zero regardless of the customer's segmentation. Therefore, $\pi_2=0$.

We next consider the case when $Pr(h)>Pr(l)$ (or equivalently, $\gamma>\frac{1}{2}$). For its low-segment customer, firm 1 offers a deterministic promotional incentive and its expected profit is zero. For its high-segment customer, firm 1 randomizes the promotional incentive following Eqs (A.11) and (A.12), and its expected profit is $E[V|h]-\overline{m}$. When $\gamma>\frac{1}{2}$ and $0\le\zeta<1$, firm 1's expected profit is

$$\pi_1=Pr(h)(E[V|h]-\overline{m})=\frac{\gamma(1-\gamma)(1-\zeta)(1+\zeta)}{(2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1}(E[V|h]-E[V|l])>0.$$

Firm 2 earns nothing from its low-segment customer in expectation, but makes a positive expected profit $E\left[V|\tilde{h}\right]-\overline{m}$ from its high-segment customer. Therefore, when $\gamma>\frac{1}{2}$ and $0\le\zeta<1$, firm 2's overall expected profit is

$$\pi_2=Pr\left(\tilde{h}\right)\left(E\left[V|\tilde{h}\right]-\overline{m}\right)=\frac{2(2\gamma-1)(1-\gamma)(1-\zeta)\zeta}{(2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1}(E[V|h]-E[V|l])\ge 0.$$

When $\zeta=0$, $\pi_2=0$. When $\zeta>0$, $\pi_2>0$. We can also verify that $\pi_1>\pi_2$.

A.6. Proof of Proposition 4

Proof. When $Pr(h)\le Pr(l)$, firm 1's expected profit is $\pi_1=\frac{\gamma(1-\gamma)(1-\zeta)}{1-\zeta+2\zeta\gamma}(E[V|h]-E[V|l])$. The first-order derivative of π_1 w.r.t. ζ is

$$\frac{\partial\pi_1}{\partial\zeta}=\frac{-2\gamma^2(1-\gamma)}{(1-\zeta+2\zeta\gamma)^2}(E[V|h]-E[V|l])<0.$$

When $Pr(h)>Pr(l)$, firm 1's expected profit is $\pi_1=\frac{\gamma(1-\gamma)(1-\zeta)(1+\zeta)}{(2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1}(E[V|h]-E[V|l])$ and firm 2's expected profit is $\pi_2=\frac{2(2\gamma-1)(1-\gamma)(1-\zeta)\zeta}{(2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1}\times(E[V|h]-E[V|l])$. The first-order derivative of π_1 and π_2 w.r.t. ζ are respectively

$$\frac{\partial\pi_1}{\partial\zeta}=\frac{2\gamma(1-\gamma)\left((3\gamma-2)\zeta^2-2\gamma\zeta+3\gamma-2\right)}{\left((2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1\right)^2}(E[V|h]-E[V|l]),$$
$$\frac{\partial\pi_2}{\partial\zeta}=\frac{2(2\gamma-1)(1-\gamma)\left((4\gamma-3)\zeta^2-2\zeta+1\right)}{\left((2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1\right)^2}(E[V|h]-E[V|l]).$$

We find that:

(1) $\frac{\partial\pi_1}{\partial\zeta}\ge 0$ if $\zeta\in[0,max\{0,\zeta_1\}]$, and $\frac{\partial\pi_1}{\partial\zeta}<0$ if $\zeta\in(max\{0,\zeta_1\},1]$ where $\zeta_1=\frac{\gamma-2\sqrt{(2\gamma-1)(1-\gamma)}}{3\gamma-2}$. $\zeta_1\le 1$ always holds.

(2) $\frac{\partial\pi_2}{\partial\zeta}\ge 0$ if $\zeta\in[0,\zeta_2]$, and $\frac{\partial\pi_2}{\partial\zeta}<0$ if $\zeta\in(\zeta_2,1]$ where $\zeta_2=\frac{1-2\sqrt{(1-\gamma)}}{4\gamma-3}$. $0\le\zeta_2\le 1$ always holds.

A.7. Proof of Proposition 5

Proof. The CDI's expected profit is $\Pi=\pi_1+\pi_2$.

When $Pr(h)\le Pr(l)$,

$$\Pi=\pi_1=\frac{\gamma(1-\gamma)(1-\zeta)}{1-\zeta+2\zeta\gamma}(E[V|h]-E[V|l])$$

The first-order-derivative of Π w.r.t. ζ is

$$\frac{\partial \Pi}{\partial \zeta}=\frac{\partial \pi_1}{\partial \zeta}<0$$

When $Pr(h)>Pr(l)$,

$$\begin{aligned}\Pi &= \frac{\gamma(1-\gamma)(1-\zeta)(1+\zeta)}{(2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1}(E[V|h]-E[V|l]) \\ &\quad+\frac{2(2\gamma-1)(1-\gamma)(1-\zeta)\zeta}{(2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1}(E[V|h]-E[V|l]) \\ &=\frac{(1-\gamma)(1-\zeta)(5\gamma\zeta-2\zeta+\gamma)}{(2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1}(E[V|h]-E[V|l]).\end{aligned}$$

The first-order-derivative of Π w.r.t. ζ is

$$\frac{\partial \Pi}{\partial \zeta}=2(1-\gamma)\frac{\left(\left(11\gamma^2-12\gamma+3\right)\zeta^2+\left(-2\gamma^2-4\gamma+2\right)\zeta+3\gamma^2-1\right)}{\left((2\gamma-1)\zeta^2+2(2-3\gamma)\zeta+1\right)^2}(E[V|h]-E[V|l]).$$

Therefore, $\frac{\partial \Pi}{\partial \zeta}\geq 0$ when $\zeta\in[0,max\{0,\zeta_0\}]$ and $\frac{\partial \Pi}{\partial \zeta}<0$ when $\zeta\in(max\{0,\zeta_0\},1]$ where $\zeta_0=\frac{-1+2\gamma+\gamma^2-2\sqrt{(2\gamma-1)(1-\gamma)(4\gamma^2+\gamma-1)}}{11\gamma^2-12\gamma+3}$. Note that $\zeta_0\leq 1$ always holds.

A.8. Proof of Proposition 6

Proof. When $Pr(h)\leq Pr(l)$, the CDI's expected profit is always decreasing in ζ. Therefore, the CDI will always choose $\zeta=0$. In another word, the CDI only serves firm 1.

When $Pr(h)>Pr(l)$, the CDI's expected profit is increasing in ζ when $\zeta\in[0,max\{0,\zeta_0\}]$ and decreasing in ζ when $\zeta\in(max\{0,\zeta_0\},1]$. The CDI will choose $\zeta=max\{0,\zeta_0\}$. $\zeta_0>0$ requires that $\gamma>\frac{\sqrt{3}}{3}$. Therefore, the CDI serves two firms and set $\zeta=\zeta_0$ when $\gamma>\frac{\sqrt{3}}{3}$. Otherwise, it only serves firm 1.

It can also be verified that $\frac{d\zeta_0}{d\gamma}>0$ when $\gamma>\frac{\sqrt{3}}{3}$.

A.9. Proof of Proposition 7

Proof. Substitute the optimal similarity factor ζ_0 into Π, we have the following expected profits.

(1) When $\gamma\leq\frac{\sqrt{3}}{3}$,

$$\zeta^*=0 \text{ and } \Pi=\gamma(1-\gamma)(E[V|h]-E[V|l]).$$

The CDI's problem of profit maximization can be represented as

$$\begin{aligned}&\max_{\psi,\phi}\ \Pi=\gamma(1-\gamma)\left(\frac{\lambda\psi}{\lambda\psi+(1-\lambda)\phi}-\frac{\lambda(1-\psi)}{\lambda(1-\psi)+(1-\lambda)(1-\phi)}\right)R \\ &s.t.\qquad \gamma\leq\frac{\sqrt{3}}{3},0\leq\phi<\psi\leq 1\end{aligned}$$

Differentiate Π w.r.t. ψ and ϕ, we have

$$\frac{\partial \Pi}{\partial \psi}=\lambda(1-\lambda)R>0,\frac{\partial \Pi}{\partial \phi}=-\lambda(1-\lambda)R<0.$$

Therefore, the CDI will choose $\psi=1$ and $\phi=0$ when $\lambda\leq\frac{\sqrt{3}}{3}$. When $\lambda>\frac{\sqrt{3}}{3}$, we let $\psi=\frac{\frac{\sqrt{3}}{3}-(1-\lambda)\phi}{\lambda}$. Differentiate Π w.r.t. ϕ, we have

$$\frac{\partial \Pi}{\partial \phi}=-(1-\lambda)R<0.$$

Therefore, the CDI will choose $\phi=0$ and $\psi=\frac{\sqrt{3}}{3\lambda}$.

The potential optimal solutions are as follows.

(a) when $\lambda\leq\frac{\sqrt{3}}{3}$, $\{\psi=1,\phi=0\}$;

(b) when $\lambda>\frac{\sqrt{3}}{3}$, $\{\psi=\frac{\sqrt{3}}{3\lambda},\phi=0\}$.

(2) When $\gamma > \frac{\sqrt{3}}{3}$,

$$\zeta^* = \frac{-1+2\gamma+\gamma^2-2\sqrt{(2\gamma-1)(1-\gamma)(4\gamma^2+\gamma-1)}}{11\gamma^2-12\gamma+3}$$
$$\text{and}\, \Pi = \frac{(1-\gamma)(1-\zeta^*)(5\gamma\zeta^*-2\zeta^*+\gamma)}{(2\gamma-1)(\zeta^*)^2+2(2-3\gamma)\zeta^*+1}(E[V|h]-E[V|l]).$$

The CDI's profit maximization problem can be represented as

$$\begin{aligned} max_{\psi,\phi}\, \Pi = & \frac{(1-\gamma)(1-\zeta^*)(5\gamma\zeta^*-2\zeta^*+\gamma)}{(2\gamma-1)(\zeta^*)^2+2(2-3\gamma)\zeta^*+1} \\ & \times\left(\frac{\lambda\psi}{\lambda\psi+(1-\lambda)\phi}-\frac{\lambda(1-\psi)}{\lambda(1-\psi)+(1-\lambda)(1-\phi)}\right)R \\ s.t. \qquad & \gamma \geq \frac{\sqrt{3}}{3}, 0 \leq \phi < \psi \leq 1 \end{aligned}$$

where $\zeta^* = \frac{-1+2\gamma+\gamma^2-2\sqrt{(2\gamma-1)(1-\gamma)(4\gamma^2+\gamma-1)}}{11\gamma^2-12\gamma+3}$.

Let $\Lambda = \frac{(1-\gamma)(1-\zeta^*)(5\gamma\zeta^*-2\zeta^*+\gamma)}{(2\gamma-1)(\zeta^*)^2+2(2-3\gamma)\zeta^*+1}\left(\frac{\lambda\psi}{\lambda\psi+(1-\lambda)\phi}-\frac{\lambda(1-\psi)}{\lambda(1-\psi)+(1-\lambda)(1-\phi)}\right)R+\mu_1(\lambda\psi+(1-\lambda)\phi-\frac{\sqrt{3}}{3})$, where μ_1 is Lagrange multiplier. From the Envelop Theorem, we have

$$\frac{\partial\Lambda}{\partial\psi} = \frac{\partial\Pi}{\partial\gamma}\frac{\partial\gamma}{\partial\psi}+\frac{\partial\Pi}{\partial\psi}+\lambda\mu_1,$$
$$\frac{\partial\Lambda}{\partial\phi} = \frac{\partial\Pi}{\partial\gamma}\frac{\partial\gamma}{\partial\phi}+\frac{\partial\Pi}{\partial\phi}+(1-\lambda)\mu_1.$$

where $\frac{\partial\Pi}{\partial\gamma} = \frac{(1-\zeta^*)\left((5\zeta^*+1)\left((2\gamma-1)\gamma(\zeta^*)^2+2(2-3\gamma)\gamma\zeta^*+\gamma\right)-(5\gamma\zeta^*-2\zeta^*+\gamma)\left((4\gamma-1)(\zeta^*)^2+2(2-6\gamma)\zeta^*+1\right)\right)(\psi-\phi)(1-\lambda)\lambda}{\left((2\gamma-1)\gamma(\zeta^*)^2+2(2-3\gamma)\gamma\zeta^*+\gamma\right)^2}R.$

$$\begin{aligned}\frac{\partial\Lambda}{\partial\psi} = & \frac{(1-\zeta^*)\left((5\zeta^*+1)\left((2\gamma-1)\gamma(\zeta^*)^2+2(2-3\gamma)\gamma\zeta^*+\gamma\right)-(5\gamma\zeta^*-2\zeta^*+\gamma)\left((4\gamma-1)(\zeta^*)^2+2(2-6\gamma)\zeta^*+1\right)\right)(\psi-\phi)(1-\lambda)}{\left((2\gamma-1)\gamma(\zeta^*)^2+2(2-3\gamma)\gamma\zeta^*+\gamma\right)^2}\lambda^2R \\ & +\frac{(1-\zeta^*)(5\gamma\zeta^*-2\zeta^*+\gamma)(1-\lambda)\lambda}{\left((2\gamma-1)(\zeta^*)^2+2(2-3\gamma)\zeta^*+1\right)\gamma}R+\lambda\mu_1 = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial\Lambda}{\partial\phi} = & \frac{(1-\zeta^*)\left((5\zeta^*+1)\left((2\gamma-1)\gamma(\zeta^*)^2+2(2-3\gamma)\gamma\zeta^*+\gamma\right)-(5\gamma\zeta^*-2\zeta^*+\gamma)\left((4\gamma-1)(\zeta^*)^2+2(2-6\gamma)\zeta^*+1\right)\right)(\psi-\phi)(1-\lambda)}{\left((2\gamma-1)\gamma(\zeta^*)^2+2(2-3\gamma)\gamma\zeta^*+\gamma\right)^2}\lambda(1-\lambda)R \\ & -\frac{(1-\zeta^*)(5\gamma\zeta^*-2\zeta^*+\gamma)(1-\lambda)\lambda}{\left((2\gamma-1)(\zeta^*)^2+2(2-3\gamma)\zeta^*+1\right)\gamma}R+(1-\lambda)\mu_1 = 0\end{aligned}$$

Considering the constraints $\psi \leq 1$ and $\phi \geq 0$, we have the following solutions:

(c) If $\mu_1 = 0$, $\{\psi = 1, \phi = 0\}$;
(d) If $\mu_1 > 0$, $\left\{\psi = 1, \phi = \frac{\frac{\sqrt{3}}{3}-\lambda}{(1-\lambda)}\right\}$.

Based on all the cases (a)–(d), for any specific λ, the optimal solution that gives the highest profit Π for the CDI is $\{\psi = 1, \phi = 0\}$.

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